1. Prove that if \( n \) is a positive integer, then \( n \) is even if and only if \( 7n + 4 \) is even.

2. Prove that if \( m \) and \( n \) are integers and \( mn \) is even, then \( m \) is even or \( n \) is even.

3. Prove or disprove that the product of a (nonzero) rational number and an irrational number is irrational.

4. Prove that the square of an integer ends with 0, 1, 4, 5, 6, or 9. (Hint: Let \( n = 10^k + l \) where \( k = 0, 1, 2, \ldots, 9 \)).

5. (a) Prove that \( 3^n < n! \) if \( n \) is an integer greater than 6.

   (b) Prove that \( n! < n^n \) if \( n \geq 1 \)

6. Prove that \( \sum_{i=1}^{n} i \cdot i! = (n + 1)! - 1 \) whenever \( n \) is a positive integer. (Recall that \( \sum_{i=1}^{n} i = \frac{n \cdot (n + 1)}{2} \).

7. Assume that a chocolate bar consists of \( n \) squares arranged in a rectangular pattern. The bar can only be broken along vertical or horizontal lines separating the squares. (Think of a Hershey’s bar.)

   Assuming that only one piece can be broken at a time, determine how many breaks you must make in order to break the bar into \( n \) squares. Use induction to prove your answer is correct.