Announcements

- HW due tomorrow
  Program

- HW is coming - due next Friday

- Midterm 2 in 1 week from next Monday (Nov. 8)
  (or later that week)
Pivot

```
begin
  13
  /\  \
 /   \ /
11   6
 \
 \  
 55
```

```
     +2
   / \  \
5   +2
 /   \ /
3   6
 \
 \  
 55
```

```
     +1
   / \  \
3   55
 /   /
13   \
/     
11
```

```
     +1
   / \  \
5   55
 /   /
13   \
/     
11
```
Binary Search Trees

A binary tree such that each internal node $v$ of $T$ stores a key $k$, and:

- keys stored at nodes in the left subtree of $v$ are less than or equal to $v$,
- keys stored in the right subtree are greater than or equal to $v$. 
Is 68 in the tree?
Q: What type of traversal will print the elements in sorted order?

17 - 28 - 29 - 32 - 44
How do we search for an element in the tree?

- Check root.
- If root < x:
  - Search right
- If root > x:
  - Search left
- Else, return yeah!

+ base case
TreeSearch\( (k, v) \):  
Input: key \( k \) to search for at a node \( v \) in tree 
Output: A node \( w \) of \( T \) s.t. either \( w \) holds key \( k \) or \( w \) is the leaf node where \( k \) would belong

```
if \( \text{key}(v) = k \)
  return \( v \)
else if \( \text{key}(v) < k \)
  return TreeSearch\( (k, v \rightarrow \text{left}) \)
else if \( \text{key}(v) > k \)
  return TreeSearch\( (k, v \rightarrow \text{right}) \)
else // at a leaf
  return \( v \)
```
How long does searching take?

$O(h)$ (h is height of tree)
Update Operations

- How do we insert?

Insert (12)
Insert (67)
Best versus worst case:

\[ 2^0 + 2^1 + \ldots + 2^h = n \]

\[ \Rightarrow 2^{h+1} - 1 = n \Rightarrow h \approx \log n \]

How deep is each tree (in terms of \( n = \# \) of nodes in tree)?