Announcements

- Posted review session info & worksheet

- Final exam next Monday at (10? noon?)
Shortest paths in a graph. (Ch 12)

Suppose we have \( G = (V, E) \) and each edge \( e \in E \) has a length \( l_e \).

Here, we'll assume \( G \) is directed: \( u \rightarrow v \).

Goal: Given two vertices, find shortest path between them.
We'll actually do something harder:

Given a source vertex $s$, compute shortest path from $s$ to every other vertex.

Greedy idea:

Start with a set $S$. (Initially $S = \{s\}$)

At each step, grow out from $S$, taking next shortest path from $S$ to a new vertex and adding that to $S$. (shortest path tree)

set of vertices where I "know" the shortest path to $s$
Greedy idea:
Start with source (here, St. Louis)
Let $S = \{s\}$
Consider edges going out from $S$.

At each step, grow out from $S$ taking next shortest path from $S$ to a new vertex $v$ and adding that to $S$. 
**Pseudo code: Dijkstra's algorithm**

(actually Leyzorek, Gray, Johnson, Ladew, Meeker, Petry + Sell)

\[ SPtree(G, s) : \]
\[ S \leftarrow \{s\} \]
\[ D[s] \leftarrow 0 \]
\[ T \leftarrow \emptyset \]

← distance array, initialized to \(\infty\)

while \(S \neq V\) do
  select node \(v\) with at least one edge into \(S\) where \(d'(v) = \min_{(u,v) \in E, u \in S} D[u] + \ell_{uv}\) is minimized
  \[ S \leftarrow S \cup \{v\} \]
  \[ D[v] \leftarrow d'(v) \]
  \[ T \leftarrow T \cup \{(u,v)\} \]
Claim: At each stage, $T$ is a set of shortest paths from $s$ to $S$.

pf: induction on $|S|$

(go take 314)
Improved Pseudo code

\[ \text{Dijkstra}(G, s) : \]

Create array \( D[v] \), initially all \( \infty \)

\[ S \leftarrow \{s\} \]

\[ D[s] \leftarrow 0 \]

for every edge \((s, u)\)

set \( D[u] \leftarrow d_{su} \)

While \( S \neq V \)

select node \( v \in S \) with \( D[v] \) minimized

\[ S \leftarrow S \cup \{v\} \]

for each edge \((v, u)\)

if \( D[v] + d_{uv} < D[u] \)

\[ D[u] \leftarrow D[v] + d_{uv} \]

\( \text{O}(\log n) \)

\( \text{O}(\log n) \)
Runtime

\[ \leq d(v) \text{ times for each vertex's value } D[v] \text{ to be modified} \]
\[ O(\log n) \text{ time each time} \]

\[ \sum_{v \in V} d(v) \log n = \log n \sum_{v \in V} d(v) \]

\[ \approx O(m \log n) = (\log n)^{2m} \]