Announcements
Remove in a BST

Several cases: Let v be our target node to delete

When is it easy?
Case 1: \( v \) is a leaf or \( v \) has only 1 child.

Ex: 

```
(14)
/  \
(16)  (14)
    /  \
   (16) (18)
       /    \
      (13)  (16) (14)
```
Case 2: v has two children

What can go in v's spot?
Ex:
Key: Next node in an inorder traversal has a valid value and can have at most one child.

Why? It can't have a left child. (Why?)
Consider this tree:

1
2
3
4
5
6
7
8
9
10

Take out a piece of paper.

Redraw it make this as good as possible.
AVL Trees

Height - Balance Property:
For every node of the tree, the heights of the children differ by at most 1.

\[ \Rightarrow \text{max height} \leq \]

(How do we calculate height again?)
Ex:
Now: How can we mess this up?
(In other words, how can the height change?)
Insert:

\[
\text{insert}(54)\]

Diagram:

```
        44
       / \
      17   78
     /     / \
    32     50  88
   /     /   /   /
  40   51  89  62
```
So: consider the lowest node which does not satisfy height-balance property \( \Xi \) - call this \( \Xi \).

Let \( \gamma \) be \( \Xi \)'s child with larger height.

Let \( \times \) be \( \gamma \)'s child with larger \( \gamma \) height.

Now - fix it!
What did you do?
Another - insert (4, 9)
So consider the lowest node which does not satisfy height-balance property U - call this

Let be t's child with larger height.

Let be y's child with larger height.

Now - fix it!
What did you do?
Generalize - Consider x, y, z, w. How can we restructure?

(Hint: What is inorder traversal of these in each case?)
Actual picture:

Where do the sub trees go??
Another

\begin{itemize}
  \item \textbf{1} \quad \textbf{2} \quad \textbf{3}
  \item \textbf{T1} \quad \textbf{T2} \quad \textbf{T3}
  \item \textbf{T4}
\end{itemize}
Any way you do this, "2" becomes the root of the new subtree with "1" to the left and "3" to the right.

What about $T_1, T_2, T_3, \& T_4$?