1. Use induction to prove that $n^n \geq n!$ whenever $n \geq 1$ is an integer.

2. Use induction to prove that

$$\sum_{k=1}^{n} k^2 = n(n+1)(2n+1)/6.$$ 

3. Use induction to prove that

$$\sum_{k=1}^{n} k \cdot k! = (n+1)! - 1.$$ 

4. Prove that for every positive integer $n$,

$$\sum_{k=1}^{n} k2^k = (n - 1)2^{n+1} + 2.$$ 

5. If $A$ and $B$ are sets, then the symmetric difference of $A$ and $B$ is written $A \triangle B$ and is defined to be $(A - B) \cup (B - A)$. Prove that if $A_1, A_2, \ldots, A_n$ are sets, then

$$((A_1 \triangle A_2) \triangle A_3) \cdots \triangle A_{n-1}) \triangle A_n = \{x : x \text{ is in an odd number of the sets } A_1, A_2, \ldots, A_n\}.$$ 

6. Assume that a chocolate bar consists of $n$ squares arranged in a rectangular pattern. The bar can only be broken along vertical or horizontal lines separating the squares. (Think of a Hershey's bar.)

Assuming that only one piece can be broken at a time, determine how many breaks you must make in order to break the bar into $n$ squares. Use induction to prove your answer is correct.