1. Let \( f_n \) be the \( n^{th} \) Fibonacci number, defined as \( f_n = f_{n-1} + f_{n-2} \) with \( f_0 = 0 \) and \( f_1 = 1 \). (Hint: Remember, induction is your friend when doing recurrences!)

(a) Prove that \( \sum_{i=1}^{n} (f_i)^2 = f_n f_{n+1} \) whenever \( n \) is a positive integer.

(b) Show that \( f_{n+1} f_{n-1} - (f_n)^2 = (-1)^n \) when \( n \) is a positive integer.

2. The set of leaves and the set of internal nodes of a full binary tree can be defined recursively, using the definition of \( T_1 \cdot T_2 \) from class or as it is presented in section 5.3 of the book.

- Basis step: The root \( r \) is a leaf of the full binary tree with one vertex. This tree has no internal vertices.
- Recursive step: The set of leaves of the tree \( T_1 \cdot T_2 \) is the union of the sets of leaves of \( T_1 \) and \( T_2 \). The internal vertices of \( T_1 \cdot T_2 \) are the (new) root as well as the union of the sets of internal vertices of \( T_1 \) and \( T_2 \).

Use induction to show that the number of leaves of a full binary tree \( T \) is one more than the number of interval vertices in a full binary tree.

3. Find a recurrence relation for the number of ways a person can climb \( n \) stairs if the person can take 1, 2, or 3 stairs at a time. (Don’t forget the base cases.) Now, how many ways are there for a person to climb 8 stairs?

4. Give exact solutions to the following recurrences. Show your work.
   (a) \( A(n) = 5A(n-1) - 4A(n-2) \), \( A(0) = 0 \), \( A(1) = 3 \).
   (b) Find the solution to the same recurrence as part (a), with \( A(0) = 3 \), \( A(1) = 4 \).
   (c) \( C(n) = -5C(n-1) - 6C(n-2) + 3 \cdot (-2)^n \), \( C(0) = 0 \) and \( C(1) = -2 \).

5. Given general form solutions to the following recurrences. (Note: this means you don’t have to solve for the constants!)
   (a) \( a_n = 8a_{n-2} - 16a_{n-4} + 3n - 2 \)
   (b) \( b_n = 8b_{n-1} - 16b_{n-2} + (n^2 - 1)4^n \)
   (c) \( c_n = 5c_{n-1} - 8c_{n-2} + 4c_{n-3} + n^3 - n^2 \)