Announcements

- Final HW not graded
- One question from final
- Sample final is printed
- Lab tomorrow
- Office hours next Friday: 9-11 am
Directed Graphs

Directed graphs are encountered in many applications.

\((u, v) \in E\) : \(\rightarrow\)

We say the number of edges going into \(u\) is the in-degree.

(And out-degree is the # of edges leaving the vertex.)
Traversals in directed graphs

Detecting if there is a path from \( s \) to \( t \) in a directed graph can be done in \( O(m+n) \) time.

Idea: Modify BFS/DFS to only add outgoing edges to stack/queue.
Directed Acyclic Graphs

If no directed cycles, called a directed acyclic graph, or DAG.

While specialized still useful:

Ex: pre-reqs in a degree program

\[
\text{cs 150 } \rightarrow \text{ cs 180 } \quad \rightarrow \text{ cs 443 } \quad \rightarrow \text{ cs 224 }
\]

\[
\text{math 135 } \quad \text{cs 443} \quad \text{cs 224}
\]
Ex: Inheritance in C++

(Compilation)

Ex: Completing a large project by breaking into smaller ones
Let $G$ be a directed graph with $n$ vertices.

A topological ordering of $G$ is a list $v_1, v_2, \ldots, v_n$ such that for every edge $(v_i, v_j) \in E$, $i < j$.

(So we order vertices so that edges only go forward.)
Not unique:
Prop: $G$ has a topological ordering if and only if it is acyclic.

\[ \Rightarrow: \text{Supps top ordering } v_1, \ldots, v_n \text{ then no cycle} \]
\[ \Leftarrow: \text{Supps acyclic.} \]
\[ \Rightarrow\text{ Some vertex has indegree 0} \]
\[ \Rightarrow\text{ Can go first} \]
\[ \Rightarrow\text{ Remove } v \text{ and repeat} \]
Algorithm:

Implement previous proof.

Find $v$ of indegree $\geq 2$.
Put it next.
Remove $v$ & its edges.
Pseudo code:

$S = \text{initially empty stack}$

For all $u \in V$

Let $I_u = \text{in-degree of } u$

If $I_u = 0$

$S._\text{push}(u)$

$i = 1$

While $!S._\text{empty}()$

$u = S._\text{pop}()$

Let $u$ be vertex $i$, $i = i + 1$

For all $(u,v) \in E$

$\begin{align*}
I_v & = I_v - 1 \\
\text{if } I_v & = 0 \text{ then } S._\text{push}(v)
\end{align*}$

$O(m)$
Claim: Yields a topological ordering

Key insight:

When $$\exists v \forall j = 0$$, all vertices with edges going into $$v$$ have already be "placed" earlier.
Runtime: $O(m+n)$