CS314: Algorithms  
Homework 2

1. Suppose you are given an array $A[1..n]$ of integers. Describe and analyze an algorithm that finds the largest sum of elements in a contiguous subarray $A[i..j]$. For example, if the array contains $(-6, 12, -7, 0, 14, -7, 5)$, the largest sum of contiguous entries is $19 = 12 - 7 + 0 + 14$.

2. We define a subsequence as anything that can be obtained from a sequence (or list of things) by extracting a subset of the elements but keeping them in the same order. For example, the strings C, YAIOAI, and DYNAMICPROGRAMMING are all subsequences of the string DYNAMICPROGRAMMING.

Let $A[1..n]$ and $B[1..n]$ be to arbitrary arrays. A common subsequence of $A$ and $B$ is another sequence that is a subsequence of both $A$ and $B$. Describe an efficient algorithm to compute the length of the longest common subsequence of $A$ and $B$.

3. Consider a graph with $n$ vertices. Recall that a subset of the vertices is called independent if no two of them are joined by an edge. Finding large independent sets is difficult in general, as we discussed in class, but can be done on some simple classes of graphs.

Call a graph a path if its vertices can be written as $v_1, v_2, \ldots, v_n$ with an edge between each $v_i$ and $v_{i+1}$ (but no other edges). With each vertex $v_i$, we associate a weight $w_i$.

Our goal in this problem is to find the largest weight independent set. (Note that this is different from the largest independent set, since here we take the weights into account!)

(a) Construct an example showing why the following simple greedy algorithm does NOT always work.

```
S ← ∅
While G is not empty:
    Pick a node $v_i$ of maximum weight
    Add $v_i$ to $S$
    Delete $v_i$ and its neighbors from $G$
Return S
```

(b) Construct an example showing why the following different simple greedy algorithm does NOT always work.

```
S_1 ← \{v_i with i odd\}
S_2 ← \{v_i with i even\}
oddsum ← sum of all weights in S_1
evensum ← sum of all weights in S_2
if evensum > oddsum
    return $S_2$
else
    return $S_1$
```

(c) Give an algorithm that takes an $n$-vertex path $G$ with weights and returns an independent set of maximum total weight. Your running time should be polynomial in $n$. 