Problems

1. Recall the makespan problem discussed in class. We discussed the fact that our greedy approximation algorithm does not always give an optimal makespan assignment, but only a 2-approximation. Given an example of a set of jobs (along with a number of machines) where the greedy algorithm fails to return a solution with optimal size.

2. Recall the shortest first greedy algorithm for the interval scheduling problem that we discussed in class: Given a set of intervals, repeatedly pick the shortest interval $I$, delete all other intervals that overlap $I$, and repeat as long as there is an interval still in the set.

In an earlier lecture, we saw that this does NOT always produce a maximum size set of non-overlapping intervals. However, it turns out to have the following interesting approximation guarantee. If $s^*$ is the maximum size of a set of non-overlapping intervals, and $s$ is the size of the set produced by our greedy shortest first algorithm, then $s \geq \frac{1}{2}s^*$, so that this greedy algorithm is a 2-approximation. Prove this fact.

3. Consider a different heuristic for constructing a vertex cover of a connected graph $G$: compute a depth first spanning tree of $G$, and return the set of non-leaf nodes in the tree. Prove that this set of vertices indeed is a vertex cover, and that it is in fact a 2-approximation.