Erratum for On the Height of a Homotopy

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Given 2 homotopic curves in a topological space, there are several ways to measure similarity between the curves, including Hausdorff distance and Fréchet distance. In [1], we examined a measure of similarity which considers the family of curves represented in the homotopy between the curves, and measures the longest such curve, known as the *height* of the homotopy. In other words, if we have two homotopic curves on a surface and view a homotopy as a way to morph one curve into the other, we wish to find the longest intermediate curve which must occur along the morphing.

Our setting was a pair of disjoint embedded homotopic curves (where the endpoints remained fixed over the course of the homotopy) in an edge-weighted planar triangulation satisfying the triangle inequality. In [1], we claimed that among minimal height homotopies between the two curves, there exists an embedded isotopy; in other words, the homotopy with minimum height would never make a "backwards" move and would result in disjoint simple intermediate curves. While we still conjecture that this holds, our proof in the appendix of [1] was not correct.

To be precise, consider a transverse orientation on a path that (locally) indicates where the path was previously. A move is considered *(locally) forward* if the move respects the transverse orientation. Figure 1 shows a forward move applied to a path where the transverse orientation is represented by shading; here, the shading is "behind" the curve, so the forward move goes away from the shaded side. Note that since this is purely local, a forward move may still cause the intermediate path to be non-simple. Also, note that move sequences consisting of only locally forward moves can have "spirals."

The problem occurs in the lemma where we stated that if there is a backwards move for a move sequence in thin position, then the move immediately prior to it must share an edge with the backwards move. Our proof claimed that if the move before the first backwards move did not share an edge, than we could swap the two moves without increasing the length of intermediate curves.

This is not true. Consider the case where the move immediately preceding the first backwards move decreased the length of the curve, and then the first backwards move increases it (such as a face shortening followed by a face lengthening). If these two moves are do not share an edge, than swapping their order will result in a longer intermediate curve, which means our move sequence is not in thin position.

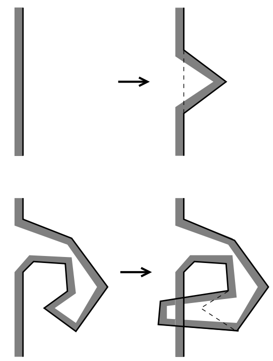

Figure 1: Two forward moves. Locally both move away from the region previous visited. Globally, the second results in a non-simple path.

While the main theorem was incorrect, we have partial results characterizing what backwards move sequences look like. For example, the following lemmas hold:

Lemma 1 *Every intermediate curve in the move sequence has algebraic self intersection number equal to zero.*

Lemma 2 *The first backwards move must occur immediately following a local minimum in the move sequence.*

References

Erin Chambers and David Letscher, "On the Height of a Homotopy", in *Proc. of the 21st Canadian Conference on Computational Geometry*, 2009.

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