For this homework, you may write up solutions with 1 partner; both of you will receive the same grade based on your joint writeup. Please make sure to read the course policies on homework before writing up your homework.

1. Show that $x^3$ is $O(x^4)$ but that $x^4$ is not $O(x^3)$.

2. Show that if $a$ and $b$ are real numbers with $a > 1$ and $b > 1$, if $f(x)$ is $O(\log_b(x))$, then $f(x)$ is $O(\log_a(x))$.

3. Give a big-O estimate (as tight as possible) of the following function. Be sure to justify your answer, either using theorems from class or a direct big-O proof.

$$f(x) = (\pi + (e^{10!}))^7 + \sum_{n=0}^{10} \left( \frac{1}{e} + n \right)^{25} x^n$$

4. Sort the following functions from asymptotically smallest to asymptotically largest. Include a proof for each relationship. To simplify your answers, write $f(n) \ll g(n)$ to mean $f(n) = O(g(n))$, and write $f(n) \equiv g(n)$ to mean $f(n) = \Theta(g(n))$.

For example, the functions $n^3$, $2n$, and $2\log_2 n$ could be sorted as $2n \equiv 2\log_2 n \ll n^3$ or as $2\log_2 n \ll 2n \ll n^3$. Further, you would need to include three proofs: that $2n = O(2\log_2 n)$, that $2n = \Omega(2\log_2 n)$, and that $2n = O(n^3)$.

$$\frac{n}{\log n}, \quad 8n^3 + 10n + 1,024, \quad n \log(n^4), \quad 2^{\log_2 n}, \quad \frac{n}{\log n}$$

5. Suppose that $f(x)$ is $O(g(x))$. Does it follow that $2f(x) = O(2g(x))$? Prove your answer.