Reminder: "I don't know policy"

- Time: Back on Monday
- A note on office hours...
- Heat up, due next Friday

Announcements

CS 314 - Graphs
A graph $G = (V, E)$ is a set whose edges (crosses) between vertices (nodes).

$V = \{ v_1, v_2, v_3 \}$

$E = \{ \{ v_1, v_2 \}, \{ v_1, v_3 \}, \{ v_2, v_3 \} \}$
Internet Back bone

Face Book

Maps

Relationships - Speed, Res.

Functions

Examples:
Definitions:

- $G$ is an unordered edge if every edge is an unordered edge.

- $G$ is directed if every edge is directional.

- $G = (V, E)$ is an ordered pair of a set of vertices $V$ and a set of edges $E$. $V = \{v_1, v_2, \ldots, v_n\}$.

Can get $x$ to $y$ if $x, y$ are connected by a path.

Diagram:

- A graph with labeled vertices $x$ and $y$.
- Edges connecting various vertices.
- A triangle with a vertex labeled $n$.
Definitions:
- A path is simple if all vertices are distinct.
- A path is simple if all edges are distinct.
- A path is simple if all vertices except the first and last occur exactly once.
- For $v_i = v_k$, if $i < k$ is a simple path.
- The degree of a vertex $v_i$ is $\delta(v_i)$. 

Note: $n$ is the number of adjacent edges. 

Simple No. Y 

\[ P = uvwx \]
Proof: Every edge counts twice, so \( |E| \) counts every edge twice. But \( \sum_{v \in V} \deg(v) \) counts every vertex \( v \) exactly \( \deg(v) \) times. Thus:

\[
\sum_{v \in V} \deg(v) = 2|E|
\]

Learning (degree-sum formula):
$d(s,t) = d(v,s)$

$G_2$ $\subseteq$ $G_1$

$\# \text{edges}$

To the length of a minimum path from $u$ to $v$, $d(u,v)$, is equal.

The distance from $u$ to $v$, $d(u,v)$, is a path from $u$ to $v$. $G$ is connected if and only if for all $u,v \in V$, there exists a path from $u$ to $v$.

$\exists m$
A tree in a graph is a vertex v

A graph with \( d(v) = 1 \) is a vertex v

A tree is a connected, acyclic graph.

Def: A tree is a connected, acyclic graph.
Then we have a cycle. If they have degree
Consider ends of $f$. 


Consider a maximum length path. Simple cycle. A path is a connected graph with no

$P_f$: A tree with at least 2 vertices has

Lemma: Every tree with $\geq 2$ vertices has
If it is not connected or cyclic, then it has k-1 edges.

Base case: # vertices = 1
# edges = 0

By induction on vertices - so on n ≥ 2
Show connected or cyclic ⇒ n-1 edges.

pf (case of 3 - after a arc extra credit)

If G has n-1 edges
Then G does not contain a cycle
and G is connected

Thus: Any a of the following imply the end:
6-leaf has \((n-1)\) edges by \(\text{Th.}\).

Since leaf can't be an any \(G\)-leaf is still connected.

6-leaf is acyclic since:

\[\text{6-leaf has } 6 \leq n \leq 7 \text{ vertices.}\]

Delete the leaf.

By proof. Lemma 2 has a leaf.

(If \(G\) is a tree)

Connect a cycle.

Ind. Step: Assume \(G\) has no vertex.
Algorithms on Graphs

Basic question: Given a graph, are there connected?