Announcements.

- HW 3 out, due on Friday (written)
- No office hours Thursday 1-2; 9-10 am instead, will be available on Thursday.
  (Also will be in Wed. morning.)
Greedy Algorithms

Main idea: Make a choice that is as good as possible in the short term.

Problem: Often doesn’t work!

Proofs of correctness are very important here!
Ex: Set of requests for a classroom.
Goal: Schedule as many classes as possible.

How many?
3
Interval Scheduling

Input: Set of requests $E = \{ I_1, \ldots, I_n \}$ in which each request starts at time $s(I)$ and finishes at time $f(I)$.

A subset of requests is compatible if no two overlap.

Goal: Find a compatible set of maximum size.
What are some “greedy” strategies?

1. Take first job, keep going ✗

2. Take shortest job ✗

3. Earliest ending time
Other ideas:

1) Take interval with fewest overlaps

Counter example
Idea: Take interval that finishes first.

```
GREEDY_SCHEDULE(S[1..n], F[1..n]):
    sort F and permute S to match
    count ← 1
    X[count] ← 1
    for i ← 2 to n
        if S[i] > F[X[count]]
            count ← count + 1
            X[count] ← i
    return X[1..count]
```

Runtime: \(O(n \log n)\)

\(O(n)\)
Correctness:

Why should this work?
(need to prove it is compatible & as large as possible.)

Compatible: X is valid since no interval is accepted if it starts before previous interval ends.
(nothing overlaps)

Notation: Let $O$ be optimal solution, $[e_{j_1}, ... , e_m]$.
Let our solution be $[e_{i_1}, ... , e_k]$.

→ Goal: $m = k$ →
Proof of correctness:

Lemma 1: For all indices $r \leq k$, $f(i_r) \leq f(j_r)$.

Induction on $r$:

Base Case: $r = 1$. Why is $f(i_1) \leq f(j_1)$?

Our alg chose job that ended earliest.

Hypothesis: Assume $f(i_{r-1}) \leq f(j_{r-1})$.

Induction Step: Consider $f(i_r) \neq f(j_r)$. We know $f(j_{r-1}) \leq s(j_r)$ since $Q$ is compatible.

$\Rightarrow f(i_{r-1}) \leq s(j_r)$ by our Hypothesis.

So our alg could have chosen $j_r$. Since our alg selected earliest available finish,
Claim: The greedy algorithm is optimal.

Proof: If it isn't then \( m > k \).

Using lemma, know \( f(i_k) \leq f(j_k) \)

Since \( m > k \), there is a \( j_{k+1} \).

\( s(j_{k+1}) \geq f(j_k) \geq f(i_k) \)

\[ \uparrow \]

We could take \( j_{k+1} \) also! \( \checkmark \)

So we have \( > k \) elements.