CS2100 - AVL Trees

Announcements

- Scholarship deadline next week
Remove: cases

- remove (12)
  - if (it has LeftChild)
  - else if (it has RightChild)
  - else (remove node)

- remove a leaf - easy!
Case 2: remove(15)

Delete and promote right child.
Case 3: 2 children: find next node in an inorder traversal, delete (20) (in sorted order)

5 + 2 = i++;

5 only has (at most) 1 child on right

Copy *it 2 into *it,

+ delete/fix promote right child
Recap: BST

Runtimes:

- **Insert**
- **Remove**
- **Find**

worst case, need to travel from root to some leaf

here: $O(n)$
Consider this tree:

Take out a piece of paper.

Redraw & make this as "good" as possible.
What did you do?
Balanced Binary Search Tree

- Red-black trees $\leq 1.2 \log n$
- Splay Trees
- AVL trees $\leq \log n$

Goal of all: $O(\log n)$
AVL Trees

Height - Balance Property: for every node of the tree, the heights of the children differ by at most 1.

\[ \Rightarrow \text{max height} \leq 2 \log_2 n \]

(How do we calculate height again?)
Ex:
Now: How can we mess this up?

(In other words, how can the height change?)

`Insert(47)`
Insert:

\[ \text{insert}(54, 47) \]

Goal: height-balance property

Fix it!
So: consider the lowest node which does not satisfy height-balance property U—call this z.

Let y be z's child with larger height.

Let x be y's child with larger height.

Now—fix it!
What did you do?
Another - insert 44

So: consider the lowest node which does not satisfy height-balance property U - call this z.

Let y be z's child with larger height.

Let x be y's child with larger height.

Now - fix it!
What did you do?
Pick middle element

Generalize - Consider $x, y, \neq z$. How can we restructure? (Hint: What is inorder traversal of these in each case?)
Actual picture:

Where do the subtrees go??

promote $y$:

pivot ($y$)
Any way you do this "2" becomes the root of the new subtree with "1" to the left and "3" to the right!

What about T1, T2, T3, and T4?
Key operation: **Pivot**

\[ \text{pivot}(b) : \text{exchange } b \text{ and its parent} \]