The Effect of (Non-)Competing Brokers on the Quality and Price of Differentiated Internet Services☆

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Abstract

Price war, as an important factor in undercutting competitors and attracting customers, has spurred considerable work that analyzes such conflict situation. However, in most of these studies, quality of service (QoS), as an important decision-making criterion, has been neglected. Furthermore, with the rise of service-oriented architectures, where players may offer different levels of QoS for different prices, more studies are needed to examine the interaction among players within the service hierarchy. In this paper, we present a new approach to modeling price competition in (virtualized) service-oriented architectures, where there are multiple service levels. In our model, brokers, as intermediaries between end-users and service providers, offer different QoS by adapting the service that they obtain from lower-level providers so as to match the demands of their clients to the services of providers. To maximize profit, players, *i.e.* providers and brokers, at each level compete in a Bertrand game while they offer different QoS. To maintain an oligopoly market, we then describe underlying dynamics which lead to a Bertrand game with price constraints at the providers' level. We also study cooperation among a subset of brokers. Numerical simulations demonstrate the behavior of brokers and providers and the effect of price competition on their market shares.

Keywords: Service-oriented architecture, Quality of Service (QoS), oligopolistic competition, service differentiation, Bertrand competition, price constraints.

1. Introduction

In today's highly competitive Internet service market, service providers, in order to survive, should offer their customers more flexibility in both their quality-of-service (QoS) and price offerings, to meet a variety of customer needs and application requirements. Clearly, any successful solution for a service provider to stay in the market, not only depends on supporting new and updated technologies, but also involves economic aspects. However, pricing the services of the network, even without considering quality differentiation, is a challenging problem that involves several issues. There have been many studies that attempted to address these issues with or without considering differentiated QoS. Pricing approaches include Paris Metro Pricing [2], congestion pricing [3, 4], rate-reliability pricing [5], and fairness pricing [6]. On the other hand, with the rise of service-oriented architectures, such as computational clouds and recursive networks [7], network virtualization such as CABO [8], and service brokerage companies such as Google's "Project Fi" [9], there is a need for more advanced solutions that manage the interactions among service providers at multiple levels. The ultimate goal in service-oriented architectures and network virtualization is to decouple the services offered by network providers from those of service providers which yield the layered structure of the network [10]. Also, brokers as the intermediaries between clients and lower-level providers, play a key role in improving the efficiency of service-oriented structures by matching the demands of clients to

☆This paper extends our preliminary model in [1] by capturing competition among more than two brokers and analyzing the effect of such competition on service qualities and prices. We also study cooperation among a subset of brokers. The text and presentation have been substantially revised.

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the services of providers. They can downgrade or upgrade a service by sharing it among customers or by combining several services to satisfy customers' demand. For example, in "Project Fi" [9], Google offers a flat data rate of $10 per gigabyte of data that is provided by either T-Mobile or Sprint, *i.e.*, Google selects the best network provider based on factors such as coverage and performance, thus adding flexibility and providing the best service to its customers. Furthermore, Project Fi customers can manage their costs based on their monthly needs. This is in contrast to network providers, *e.g.*, T-mobile and Sprint, which offer their customers fixed data plans regulated by a static contract.

In this paper, we propose and dissect with extensive numerical simulations a multi-layer network market model in which service brokers and service providers compete at different levels in an oligopoly to maximize their profit. In our settings, brokers can pay a cost to upgrade or downgrade the service that they buy from (lower-level) providers so as to offer a new service to the market (customers). The broker incurs costs when adapting a lower-level service as it expends resources to either enhance the service extended to its customers (*e.g.*, by employing delay-jitter reduction or capacity allocation techniques over a best-effort service) or degrade it (*e.g.*, by multiplexing client demands over a guaranteed service). We consider the competition among providers and among brokers separately, while brokers impose some preference constraints on (infrastructure, cloud or service) providers. We also consider conditions that may lead to a monopoly market and study how players act under such conditions. We model service quality differentiation after Hotelling's location model [11], where firms compete and price their products in only one dimension, geographic location. In our model, brokers and (lower-level) providers compete and price their services based on the quality of the service that they offer. Our numerical results show that more service differentiation generally yields more profit for all players. However, besides quality differentiation, the cost that brokers undergo also plays an important role and they should forgo maximum differentiation to reduce the cost, which leads to higher profit. Also, as the number of brokers increases, the market gets more competitive and prices drop further.

We start with the assumption that players are completely non-cooperative. This profit-seeking nature of players leads to selfish behaviors that may have negative consequences and reduce their profits. So, it is reasonable to assume that a subset of players discuss possible cooperative strategies, form coalitions, and take actions that are beneficial to all members of the group. Coalitional games have been widely explored in different disciplines such as economics and political sciences. Recently, cooperation has emerged as a new strategy that has a huge impact on improving performance from the physical layer [12, 13] up to the network layer [14]. The application of cooperative game theory in network studies has mostly focused on the traffic routing problem, network traffic engineering problems, and network connectivity problems [15, 16, 17, 18, 19]. In this paper, we also consider a two-layered market in which a subset of the brokers cooperate with each other rather than compete. Specifically, a new broker entering the market cooperates with one of the existing brokers in competition with the other broker(s). We study the impact of this cooperation on the quality that the new broker chooses, the other brokers' prices, and also customers' utility. While in most situations, collaboration improves the cooperating brokers' profit but with a negative impact on customers' utility, there are cases where both coalition brokers and customers benefit from the cooperation. In these cases, cooperation of two brokers divides the demand between the service providers in such a way that causes tougher competition, and consequently leads to lower prices, at the service providers' level.

1.1. Contributions and Paper Outline

We start by reviewing existing literature and positioning our work. We then introduce a novel *two-layered network market* model in which providers and brokers offer *differentiated services* and compete in a non-cooperative game at each layer (Section 3). We model the price selection based on the Hotelling's location model [11], and we characterize the competitive behavior of players at each level of the service hierarchy based on a Bertrand game. We consider the market at the Nash Equilibrium point, where all players are in their steady state and solve the model using a two-stage procedure. We also analyze the actions of players under a monopoly setting. Our main results, obtained with an analytical analysis and with numerical simulations, show that, *when there are only two brokers, a higher quality differentiation leads to higher provider's profit* (Section 4). We also find that, when there are multiple brokers, the cost of converting the quality becomes an important factor for profit maximization (Section 4.4). We proceed with considering cooperation among a subset of brokers in Section 5. In a non-cooperative game, all players try to maximize their own profit independently. When a new broker enters the market, the competition gets more intense and there is a large drop in the profit of existing brokers. In Section 5, we study the market where the entering broker cooperates with one of the existing brokers. We compare the profit and price of brokers (and service providers) with those achieved under the non-cooperative game. We also consider the effect of cooperation on the customers' welfare. Finally, Section 6 concludes the paper.

2. Related Work

Game theory has been applied to a wide range of networking problems to capture the interaction of (selfish or cooperating) players seeking a maximum value for their (private) utility. The assumption is that every step (or move) toward the maximization of such utility impacts the utility of other players in the model (or game). Given the connectivity nature of a network of agents, a wide range of networking mechanisms have been modeled with game theory; from the physical ISO-OSI layer with transmission power utility games [20] or spectrum sharing [21, 22] to Medium Access Control [23] to routing and packet forwarding [24, 25], both in wireless [26] and wired [27] scenarios.

Aside from modeling multi-agent protocol behaviors and the various resource allocation mechanisms, markets and pricing equilibrium have further exemplified the synergy between game theory (and economics) and networked (cloud) systems [28, 29]. In particular, network economics has been a very active research area in which both pricing and market regulation strategies have been studied widely. However, the exponential growth of Internet services in hierarchical (*i.e.*, multi-layer) markets requires a deeper study of new market features that will become available. One of the earliest work on layered networks [30], identifies and discusses some difficult economic problems related to resale and complexity of competition among multiple owners of physical networks. The authors study some integrated and unintegrated telecommunication companies and the services that they offer to create differentiated products to cover their costs. The paper does not suggest any specific architecture or policies for pricing as we do, but discusses the need for a full economic model that features oligopolistic competition among a few large companies that invest in the physical infrastructure as well as firms at the virtual network level.

Pricing for single-level games has been studied extensively. He and Walrand [31] consider a self-regulated service model, where market demand determines the service quality, *i.e.*, higher demand causes more congestion and consequently less quality. Unlike ours, in their model there is a single Internet Service Provider (ISP) who offers two classes of service with different prices to manage congestion. They show that when the price does not match the service quality, the system may end up in an equilibrium similar to the Prisoner's Dilemma game. Shetty *et al.* [22] compare the revenue of a monopolist operator with and without service differentiation. They show that the revenue is higher when an operator offers two different services. Both Li *et al.* [32] and Fulp and Reeves [33] provide a traffic- sensitive pricing scheme for differentiated network services. The focus of [33] is on maximizing the profit of the service provider who buys a differentiated service connection from domain brokers and sells it to users, whereas [32] focuses on providing economic incentives to users so as to maintain a given level of traffic load.

Two-level games have also been studied more recently. Our work is inspired by Zhang *et al.* [34] and Nagurney and Wolf [35]. They propose an economic model for the interaction and competition among service providers, network providers and users. Both studies develop a two-stage (Stackelberg) game, where service providers compete in a Cournot game, and network providers compete in a Bertrand game. In [35], the authors generalize the market of [34] by considering different demand markets served by any number of service providers and any number of network providers in which network providers offer different levels of service quality. Although our work shares the same two-level game approach with [34] and [35], in our framework we consider users and providers at each level (viewed as "users" of lower-level providers), having service preference based on quality and price, where at each level providers compete in a Bertrand game (*i.e.*, competition on price). Also, Zhang *et al.* [34] study a market with two service providers and two network providers offering the same level of service quality. Our model, however, is more realistic as we consider a market where players at both levels may offer different qualities of service.

Different game-theoretic models for differentiated service markets of users and service providers have also been proposed [36, 37, 38, 39]. In [38], the authors propose a game-theoretic model where service providers compete with duration-based contracts for differentiated service. On the other hand, the authors in [37] consider a joint price-quality market with a Stackelberg game where providers are leaders and users are followers. In their model, providers consider the migration of users when setting their price and quality. In another study, Semret *et al*. [39] consider a retail market where, for each network, three types of players interact: a service provider, a broker and a set of end-users; their main contribution is a decentralized auction-based bandwidth pricing for differentiated Internet services. They show that Progressive Second Price[1] provides a stable pricing in a market where service providers receive most of the profits, and the brokers' profit margin is small. Finally, the authors in [36] study a congestion-prone market with usage-based pricing. They propose a model for users' preference over their value and sensitivity to congestion, and based on such model they characterize the market share and optimal price for providers.

Our model considers multi-layer differentiated service games where the service obtained from the lower level can be upgraded or downgraded, and hence can be sold to the higher level provider. In our analysis, we apply price constraints when players' optimal price would lead to losing market share, and we also give insights on how players should then update their price.

We also consider cooperation among a subset of brokers in our model. Cooperative games were introduced in the 1940s [41], and are considered an important branch of game theory. Since then, many solutions for these games have been proposed [42, 43, 44, 45]. Although the Internet is considered as a set of autonomous agents in game theoretic studies, there are studies that consider coalitions among players and study the effect of cooperation on the problem at hand. Concepts and principles from cooperative game theory have enriched our understanding of resource allocation in wireless networks [46, 47, 48, 49], spectrum sharing among users [50, 51], and transmission at the physical layer [52, 53].

[1]PSP is a natural generalization of second-price auctions in the case of sharing an arbitrarily divisible resource [40].

Another line of work studies the effect of cooperation among content providers and network service providers, and the profit sharing mechanism, on resource pricing [54, 55, 56, 57]. Most of these studies either use Shapley value [58, 59] or Nash bargaining game [60] to model cooperation in network resource pricing. Shapley value emphasizes revenue distribution based on weighted marginal contribution of each entity in a group, while Nash bargaining emphasizes the Pareto optimal property and symmetry. In [54], network users are assumed to have the same preference, and therefore the pricing problem degenerates to a game between a single user and an ISP. The authors show that Nash bargaining makes the system converge to the Pareto optimal point. The authors in [56, 61] study the economics of traditional transit providers and content providers and apply cooperative game theory to find an optimal settlement between these entities. They use Shapley value profit distribution for a better engineered Internet. In [55], price theory is used to design a peer-assisted content distribution system that manages ISP resources more efficiently. The authors in [62] consider the interaction among ISPs at different levels – local ISPs and transit ISPs – and show that for local ISPs, there exists an optimal scenario where all ISPs peer with each other and jointly maximize their profit. The authors in [57, 63] examine the interplay between traffic engineering and content distribution, and study the relation between content providers (CP) and network service providers. They show how ISPs and CPs can cooperate, and why such cooperation not only guarantees a fair profit distribution among providers, but also helps improve the economic efficiency of the network system. In our study, only a subset of players (brokers) cooperate with each other and they are focused on maximizing their own total revenue while competing with the rest of players.

3. Model and Solution

In this section, we present our model and analysis of a two-level game configuration and focus on the competition among providers and brokers and what emerges as pricing of their services. Figure 1 illustrates the game-theoreftic model: At the lower level, we have two service providers, while at the higher level, we have $m \geq 2$ service sellers or brokers that deal directly with users. Note that owning a network infrastructure is expensive, and only a few large companies can afford its cost. There are however many brokerage companies. Our model's goal is limited to analyzing and understanding the dynamics of a market in a formal economic setting. To this aim, we start by considering only two network / lower-level providers in a simple oligopoly market competition. The exclusion of more complex relationships that may exist in real markets keeps our model tractable while still producing interesting results and insights.

To model service quality differentiation, as in [11, 64], we model the difference between products as differences in a product's location in a product space. The idea is widely used for both location problems [65, 66, 67] and quality differentiation [68, 69, 70] in network studies. In the Hotteling's model [11], there are two firms selling identical goods along a street. Customers are assumed to be uniformly distributed in the (geographical) space, and the transport cost is a linear function of their distance to the selected firm. A consumer selects the firm that minimizes her cost of transportation to buy the product. Hotteling concluded that two firms would locate close to each other near the center. Later, D'Aspremont *et al.* [64] changed the utility function from linear to quadratic form, leading to firms choosing to maximize their distance to the opposite player, and reaching equilibrium for price competition. Brenner [71] extended the game with the quadratic cost function to more than two firms. He has shown that for more than two firms, the "principle of maximum differentiation" does not hold, and corner firms would benefit from moving marginally toward the market center. In our work, we model service quality differentiation after Hotteling's product differentiation, where customers have different preference for service quality that is modeled by their willingness to pay for that quality.

We start by presenting our notation and some basic settings, then we discuss some analytical and numerical results.

3.1. Model Description

Let us consider a system with a continuum of customers, m service sellers (brokers), denoted by $B_i,\ i = 1, \ldots, m$, and two service providers, $S_j,\ j = 1, 2$. We assume that customers have different preference for quality (utility) described by:

$$\theta q - p$$

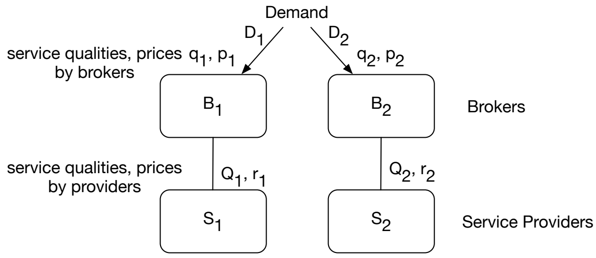

Figure 1: Game-theoretic Model of a Two-level Competition among Brokers and Service Providers. At each level, brokers and service providers are in a Bertrand competition with each other. The set of strategies are the service prices and each player tries to maximize her revenue.

where θ is the customer's marginal willingness to pay for quality q, and p is the price of service. There is a distribution of θ among customers. For simplicity, we assume that θ is uniformly distributed on an interval $\theta \in [\theta_{min}, \theta_{max}]$ and $\theta_{max} > 2\ \theta_{min}$. Customers seek a broker that maximizes their utility.

Both brokers and service providers can offer services with different qualities, but in this initial model we assume that each player only offers one class of quality [72]. The service quality offered by brokers is denoted by q_i and lies in an interval $q \in [q_{min}, q_{max}]$. The quality offered by service providers is denoted by Q_j. Also, we assume that brokers and service providers compete in an imperfectly competitive market. *Furthermore, we assume that there is no supply constraint and so there are enough resources to meet each demand.* We also assume that there is no geographical or performance limitation on service providers and that brokers can always get all their required services from the service provider that is more economically convenient. As we mentioned earlier, in our model, service providers have already incurred the cost of setting up their infrastructure, so they intend to attract part of the market and stay in the market. So without loss of generality, we assume that service provider S_1 attempts to keep at least the broker with the lowest quality (B_1) as her buyer, and service provider S_2 attempts to keep the broker with the highest quality (B_m) as her buyer (unless as we note in Section 3.6, the market does not support this assumption), while other brokers choose the service provider that offers the lower cost.

We assume that providers, and brokers, compete separately with each other in a Bertrand game. In this market structure, the players compete with each other non-cooperatively to achieve their objectives (*i.e.*, maximize profit) by controlling the price of the their services. The decision of each player is influenced by other players' actions and the action of a player may be observed by all other players. The **players** are service providers at the lower level and brokers at the higher level. The **strategy** of each player is the non- negative service price. The **payoff** (utility function) is the profit generated by selling the services. This game has a Nash equilibrium. [2] We solve the model using a two-stage procedure. First, given the service providers' price, r_i's, and demand as a function of the brokers' price, the brokers compete in a Bertrand game. The Nash equilibrium of the Bertrand game leads to an *optimal price* for the brokers and therefore the demand becomes a function of the qualities and the service providers' price r_i's. In the second stage, service providers compete in a Bertrand game to maximize their profit by setting their price. Substituting r_i's into the demand obtained by the Nash equilibrium at the previous stage, we can determine the final optimal price for the service providers and brokers. We describe the game in detail next.

3.2. Demand Distribution

Brokers first choose the quality of service that they will provide to customers, then they compete on prices. If the brokers choose the same quality, then the customers decide only based on the price, and this leads to a Bertrand competition with identical goods, whose prices should be set equal to costs, and no one makes profit. Thus the brokers should choose to offer different service qualities to make profits. Without loss of generality, we assume that there exists a strict order on quality values, that is, $q_m > ... > q_2 > q_1$, and also $Q_2 > Q_1$. Therefore, customers with a high willingness to pay for quality will buy from B_m, while customers with a low willingness will buy from B_1.

For simplicity, first let us assume that we have two brokers, B_1 and B_2. We can characterize the demand for each broker by identifying the customers who are indifferent between the two differentiated qualities. The indifferent customers, represented by θ^*, satisfy:

$$\theta^* q_1 - p_1 = \theta^* q_2 - p_2 \Leftrightarrow \theta^* = \frac{p_2 - p_1}{q_2 - q_1} \quad (1)$$

Having uniformly distributed θ, the demand for each broker, B_1 and B_2, is given by:

$$\begin{aligned} D_1(p_1, p_2) &= \frac{\theta^* - \theta_{min}}{\Delta\theta} = \frac{1}{\Delta\theta}\left(\frac{p_2 - p_1}{q_2 - q_1} - \theta_{min}\right) \\ D_2(p_1, p_2) &= \frac{\theta_{max} - \theta^*}{\Delta\theta} = \frac{1}{\Delta\theta}\left(\theta_{max} - \frac{p_2 - p_1}{q_2 - q_1}\right) \end{aligned} \quad (2)$$

where $\Delta\theta \equiv \theta_{max} - \theta_{min}$.

For more than two brokers, we can generalize Equation (1) to find indifferent customers θ_i^* between any two brokers B_i and B_{i+1}:

$$\theta_i^* q_i - p_i = \theta_i^* q_{i+1} - p_{i+1} \Leftrightarrow \theta_i^* = \frac{p_{i+1} - p_i}{q_{i+1} - q_i} \quad (3)$$

Consequently, the demand for each broker is given by:

$$\begin{aligned} D_1(p_1, p_2, \ldots, p_m) &= \frac{\theta_1^* - \theta_{min}}{\Delta\theta} \\ D_i(p_1, p_2, \ldots, p_m) &= \frac{\theta_i^* - \theta_{i-1}^*}{\Delta\theta} \qquad 1 < i < m \\ D_m(p_1, p_2, \ldots, p_m) &= \frac{\theta_{max} - \theta_{m-1}^*}{\Delta\theta} \end{aligned} \quad (4)$$

Note that in the above equations D_i's assume values in the interval $[0, 1]$. This means that if for broker B_i the demand D_i is negative, then B_i is "out of the market"; more precisely, we can rewrite the demand function as:

$$D_i = \min\left\{\max\left\{0, \frac{\theta_i^* - \theta_{i-1}^*}{\Delta\theta}\right\}, 1\right\} \text{ for } 1 \le i \le m \quad (5)$$

3.3. Brokers' Profits

Now that we have the demand distribution, we can calculate broker i's profit, assuming that converting Q_j to q_i (whether to upgrade or downgrade the service) has a marginal cost c_i:

$$\Pi_i = p_i D_i - \frac{q_i D_i}{Q_j} r_j - c_i D_i (Q_j - q_i)^2 \quad (6)$$

where r_j is the price of service that broker B_i pays to service provider S_j, and $\frac{q_i D_i}{Q_j}$ is the amount of

[2] By definition, the *Nash equilibrium* of a game is a strategy profile (list of strategies, one for each player) with the property that no player can increase her payoff by choosing a different action, given other players' actions [73].

service that B_i needs to buy to supply its own market. This is because, if we consider that the quality of the service is given by the quantity of needed resources, such as bandwidth or memory, then the required resources that a broker needs to buy can be obtained from $\frac{q_i}{Q_j} D_i$. For example, consider a broker's QoS requirement of 10Mbps (q_i), and a service provider offering 5Mbps (Q_j) channels; this would result in $(\frac{q_i}{Q_j}) = \frac{10}{5} = 2$ demand requests from the broker to the service provider to combine two provider's channels and upgrade the lower-level service. On the other hand, if q_i= 5Mbps and Q_j= 10Mbps, then this results in 0.5 demand request and possibly lower cost for the broker. We also assume that the cost to the broker, c_i, to convert the service quality that such a broker gets from the service provider, is proportional to the square of the difference in quality, $(Q_j - q_i)$. Intuitively, the cost increases more rapidly as the service quality increases, or alternatively, there is a diminishing return in service quality as more resources are allocated and cost increases. For simplicity, we assume that $c_i = c$.

Since we assume that each broker buys just from one lower-level provider that yields less cost for the broker, the following result holds:

Theorem 1. *(Oligopoly of Service Providers)*

Let us consider a market with two service providers and multiple brokers. Let us assume that the two providers offer their services in the same geographical area. To guarantee an oligopoly market at the service provider level (which captures the attempt of service providers to stay in the market), the broker with lowest quality buys from the lower quality provider and the highest quality broker buys from the higher quality provider.

Proof. We need to show that if the broker with the lowest quality B_1 prefers to buy from the higher quality provider S_2, then the other brokers also prefer to buy from S_2; therefore no broker will buy from the lower quality provider S_1, that is, we have a monopoly market at the provider level.

Let us assume that B_1 prefers to buy from S_2, then the cost of buying from S_1 must be greater than the cost of buying from S_2:

$$\frac{q_1}{Q_1} r_1 + c(q_1 - Q_1)^2 > \frac{q_1}{Q_2} r_2 + c(q_1 - Q_2)^2$$

After expanding the quadratic terms and simplifying, we have:

$$q_1 (\frac{r_1}{Q_1} - 2cQ_1) + cQ_1^2 > q_1 \left(\frac{r_2}{Q_2} - 2cQ_2 \right) + cQ_2^2$$

Grouping the terms that have q_1 as a factor, we have:

$$q_1 \left(\frac{r_1}{Q_1} - 2cQ_1 - \frac{r_2}{Q_2} + 2cQ_2 \right) > c(Q_2^2 - Q_1^2)$$

Giving that the quality of other brokers is higher than B_1 ($q_i > q_1$), the derived inequality holds for other brokers as well. Even for other brokers, buying from S_1 is more costly, therefore we have a monopoly market (no broker buys from $S1$). The same logic applies if B_m prefers to buy from S_1. Hence we prove the claim. □

Now that we have the profit function for brokers, we can find the optimal price for them. In the first stage, given the service prices r_j, and service qualities Q_j, the brokers compete in a Bertrand game with differentiated goods. We present the results for the case $m = 2$, but all results can be similarly calculated for general cases with more than two brokers. As we have seen, in a Bertrand game, players control the price to maximize their profit. The solution to the Bertrand game is hence a Nash Equilibrium, which is obtained as follows: We substitute Equation (2) into Equation (6), and solve $\partial \Pi_i / \partial p_i = 0$ to obtain Nash equilibrium, that leads to (see Appendix for the detailed derivation):

$$\begin{aligned} p_1 = \frac{1}{3}((q_2 - q_1)(\theta_{max} - 2\theta_{min}) + \frac{2q_1 r_1}{Q_1} + \\ \frac{q_2 r_2}{Q_2} + 2c(q_1 - Q_1)^2 + c(q_2 - Q_2)^2) \end{aligned} \tag{7}$$

$$\begin{aligned} p_2 = \frac{1}{3}((q_2 - q_1)(2\theta_{max} - \theta_{min}) + \frac{q_1 r_1}{Q_1} + \\ \frac{2q_2 r_2}{Q_2} + c(q_1 - Q_1)^2 + 2c(q_2 - Q_2)^2) \end{aligned} \tag{8}$$

Now the brokers' prices, p_1 and p_2, are a function of the brokers' and providers' service qualities, and providers' prices r_j's. The next step is to plug them into D_i's to obtain the demand as a function of r_j's:

$$\begin{aligned} D_1 = \frac{1}{3\Delta\theta}(\theta_{max} - 2\theta_{min}) + \\ \frac{\frac{q_2 r_2}{Q_2} - \frac{q_1 r_1}{Q_1} - c(q_1 - Q_1)^2 + c(q_2 - Q_2)^2}{3\Delta\theta(q_2 - q_1)} \end{aligned} \tag{9}$$

$$D_2 = \frac{1}{3\Delta\theta}\left(2\theta_{max} - \theta_{min}\right) + \frac{\frac{q_1 r_1}{Q_1} - \frac{q_2 r_2}{Q_2} + c\left(q_1 - Q_1\right)^2 - c\left(q_2 - Q_2\right)^2}{3\Delta\theta\left(q_2 - q_1\right)} \quad (10)$$

Now, D_1 and D_2 are dependent on service providers' prices r_j's, which shows the interaction between the two layers. Providers' prices affect the cost for brokers and in turn affect the price of brokers and consequently the demands of both brokers and providers. In the next subsection we show how to find the optimal r_j's.

3.4. Modeling Providers' Profits

At this stage, we have the total demand served by (service sold by) each broker. To have an imperfectly competitive market at the level of service providers, the combination of their price and quality should be such that each broker prefers a different service provider. Assuming B_1 prefers S_1 and B_2 prefers S_2, the following inequalities should hold for B_1 and B_2, respectively:

$$\begin{aligned} \frac{q_1}{Q_1} r_1 + c(q_1 - Q_1)^2 &< \frac{q_1}{Q_2} r_2 + c(q_1 - Q_2)^2 \\ \frac{q_2}{Q_2} r_2 + c(q_2 - Q_2)^2 &< \frac{q_2}{Q_1} r_1 + c(q_2 - Q_1)^2 \end{aligned} \quad (11)$$

These constraints ensure that broker B_1 chooses provider S_1 and B_2 chooses S_1, as the cost is lower than that of getting service from the other provider. Later we will discuss the situation when one of these constraints is violated.

For the general case, we assume that the first k brokers choose provider S_1 and the remaining brokers B_{k+1} to B_m choose S_2. Constraints (11) should hold for B_k and B_{k+1} instead of B_1 and B_2.

In this stage of the game, service providers compete in another Bertrand game. The profit of each provider is defined as:

$$\begin{aligned} U_1 &= \sum_{i=1}^{k} \frac{D_i q_i}{Q_1}(r_1 - f_1) - eQ_1^2 \\ U_2 &= \sum_{i=k+1}^{n} \frac{D_i q_i}{Q_2}(r_2 - f_2) - eQ_2^2 \end{aligned} \quad (12)$$

where eQ_j^2 is the cost of providing quality Q_j, r_j is the service price and f_j represents some general cost (fee). After plugging Equations (9) and (10) into the providers' profit, we obtain quadratic equations in r_j. To obtain the optimal solution (Nash equilibrium), we solve $\partial U_j / \partial r_j = 0$ which, for two providers, yields:

$$\begin{aligned} r_1 = {} & \frac{2f_1}{3} + \frac{f_2 q_2 Q_1}{3q_1 Q_2} + \frac{Q_1}{3q_1} \times \\ & [c(q_2 - Q_2)^2 - c(q_1 - Q_1)^2 - \\ & (q_1 - q_2)(4\theta_{max} - 5\theta_{min})] \end{aligned}$$

$$\begin{aligned} r_2 = {} & \frac{2f_2}{3} + \frac{f_1 q_1 Q_2}{3q_2 Q_1} + \frac{Q_2}{3q_2} \times \\ & [c(q_1 - Q_1)^2 - c(q_2 - Q_2)^2 - \\ & (q_1 - q_2)(5\theta_{max} - 4\theta_{min})] \end{aligned}$$

By substituting r_j's in Equations (7) and (8), we get the final values for p_i's as functions of only user preferences and service qualities (besides marginal costs/fees):

$$\begin{aligned} p_1 = {} & \frac{1}{9}\left(5c(q_1 - Q_1)^2 + 4c(q_2 - Q_2)^2\right) + \\ & \frac{4f_2 q_2 Q_1 + 5f_1 q_1 Q_2}{9Q_1 Q_2} + \frac{1}{9}(q_2 - q_1)(16\theta_{max} - 20\theta_{min}) \end{aligned}$$

$$\begin{aligned} p_2 = {} & \frac{1}{9}\left(4c(q_1 - Q_1)^2 + 5c(q_2 - Q_2)^2\right) + \\ & \frac{5f_2 q_2 Q_1 + 4f_1 q_1 Q_2}{9Q_1 Q_2} + \frac{1}{9}(q_2 - q_1)(20\theta_{max} - 16\theta_{min}) \end{aligned}$$

We obtain the final values for D_i's from Equations (9) and (10):

$$\begin{aligned} D_1 = {} & \frac{1}{9\Delta\theta}(4\theta_{max} - 5\theta_{min}) + \\ & \frac{c(q_1 - Q_1)^2 - c(q_2 - Q_2)^2}{9\Delta\theta(q_1 - q_2)} + \frac{-f_2 q_2 Q_1 + Q_2 f_1 q_1}{9\Delta\theta(q_1 - q_2)Q_1 Q_2} \end{aligned}$$

$$\begin{aligned} D_2 = {} & \frac{1}{9\Delta\theta}(5\theta_{max} - 4\theta_{min}) + \\ & \frac{c(q_2 - Q_2)^2 - c(q_1 - Q_1)^2}{9\Delta\theta(q_1 - q_2)} + \frac{f_2 q_2 Q_1 - Q_2 f_1 q_1}{9\Delta\theta(q_1 - q_2)Q_1 Q_2} \end{aligned}$$

3.5. Positive Utility

In the previous setting we assumed that customers buy service from either B_1 or B_2, even if their utility is negative. In this subsection we remove this assumption by solving a game with only positive utility customers, *i.e.*, customers whose value of $\theta q - p$ is positive. Therefore, customers with zero utility provide a lower bound on θ (we call it θ_0), which can be found by solving $\theta_0 q_1 - p_1 = 0$.

Thus θ_{min} is replaced by $\frac{p_1}{q_1}$:

$$D_1(p_1, \ldots, p_m) = \frac{\theta_1^* - \theta_0}{\Delta\theta} = \frac{1}{\Delta\theta}\left(\frac{p_2 - p_1}{q_2 - q_1} - \frac{p_1}{q_1}\right) \quad (13)$$

As in our previous setting, this is a two-stage Bertrand game, and the Nash equilibrium for each game is found by replacing the D_i's into the profit functions and solving $\partial\Pi_i/\partial p_i = 0$ and $\partial U_i/\partial r_i = 0$. We discuss the difference between this positive utility game and the previous (unconstrained utility) game later in Section 4.

3.6. Game with Constraints

At the lower level of service providers, the constraints (11) are not considered while calculating the equilibrium points. Therefore, in some situations, one of the constraints might be violated. Let us assume that after finding r_i's, the constraint for B_1 is violated, *i.e.*, $\frac{q_1}{Q_1}r_1 + c(q_1 - Q_1)^2 \geq \frac{q_1}{Q_2}r_2 + c(q_1 - Q_2)^2$. This means that, under this condition, broker B_1 incurs more cost to buy service from provider S_1 than provider S_2; so if provider S_1 does not change its price, B_1 will get service from S_2, and this situation leads to a monopoly market at the providers' level.

To find an optimal point that also meets the constraints (11), provider S_1 should set its price such that

$$r_1 < \frac{Q_1}{q_1}\left(\frac{q_1}{Q_2}r_2 + c(q_1 - Q_2)^2 - c(q_1 - Q_1)^2\right)$$

In response, provider S_2 updates its price by plugging r_1 into $\partial U_2/\partial r_2 = 0$ which leads to $r_2 = \mathcal{F}(r_1)$, *i.e.*, r_2 as a function of r_1. Thus, S_1 can replace r_2 with $\mathcal{F}(r_1)$ in its inequality to calculate an optimal price that satisfies the constraint:

$$r_1 = \frac{Q_1}{q_1}\left(\frac{q_1}{Q_2}\mathcal{F}(r_1) + c(q_1 - Q_2)^2 - c(q_1 - Q_1)^2\right) - \epsilon$$

$$\epsilon > 0$$

In this stage of the game, S_1 should find a positive value for ϵ that maximizes its profit. By substituting r_1 and r_2 as functions of ϵ, U_1 is a decreasing quadratic function of ϵ. Solving $\partial U_1/\partial\epsilon = 0$ results in optimal ϵ. If $\epsilon < 0$, it can be replaced with a small positive number close to zero. Since U_1 is decreasing with respect to ϵ, any other positive value larger than the chosen ϵ leads to less profit. Clearly, the new set of prices for the service providers is an equilibrium point for the game, since it maximizes the revenue of both providers while meeting the constraints, so each service provider does not lose its market (*i.e.*, one of the two brokers stays as its customer); therefore neither of the service providers has an incentive to change its price independently.

4. Numerical Analysis

In this section we present some numerical results to illustrate the effect of choosing different qualities of service by brokers. We consider settings with two, three and four brokers. We also study the positive game model for two brokers. We show in detail how the best strategy for any broker is to choose a quality level that maximizes quality differentiation with other brokers. Also, when there are more brokers, the higher competition leads to more reasonable prices and a lower probability of a monopoly market. In the following subsections, we start with our main observations followed by a detailed analysis of our results. Though we have obtained results for a wide range of parameters' values, we only present in this paper a representative set of these results.

4.1. Two Brokers

Observation 1. *All players (brokers and providers) make more profit as the gap between qualities of service offered by brokers increases,* i.e.*, the maximum differentiation principle applies.*

Observation 2. *When the qualities of service offered by brokers are close to each other ($\Delta q \approx \frac{\Delta p}{\theta_{max}}$) the demand mostly goes to the lower quality service. In this situation, it is more likely that monopoly happens at the service provider level.*

For the two brokers case, we consider a setting where $\theta_{max} = 1.5$, $\theta_{min} = 0.2$, $c = 0.1$, and $f_i = .01265 \times Q_i^{1.5}$. The service qualities of the providers are set to $Q_1 = 20$ and $Q_2 = 45$. For the brokers, q_2 varies between 30 and 60, and we set q_1 to different values such that it is less than, equal to, or larger than Q_1 to see how the market changes under different conditions, although here we show plots for only two different values of q_1. Figure 2 shows the results when B_1 downgrades the quality of service obtained from S_1 ($q_1 = 13$), whereas Figure 3 shows the results when B_1 upgrades that

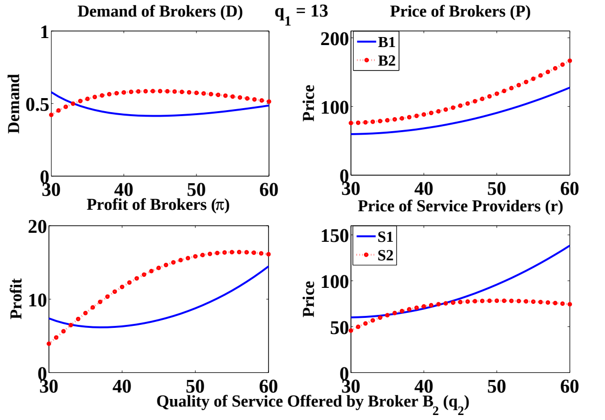

Figure 2: Price, profit and demand distribution for brokers, and price of providers, B_1 downgrading the quality, Q_1 = 20, q_1 = 13, Q_2 = 45, $30 \leq q_2 \leq 60$, as the service quality offered by broker B_2 changes.

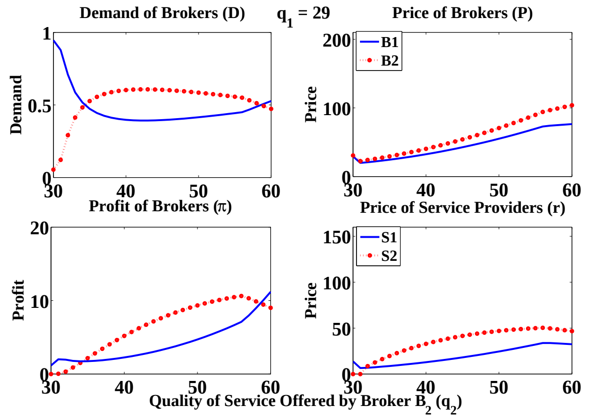

Figure 3: Price, profit and demand distribution for brokers, and price of providers, B_1 upgrading the quality, Q_1 = 20, q_1 = 29, Q_2 = 45, $30 \leq q_2 \leq 60$, as the service quality offered by broker B_2 changes.

quality ($q_1 = 29$). First, we note that the total demand constitutes the whole market. So, when the demand for one broker/provider side decreases, the demand for the other side increases, and vice versa. But this is not the case for prices and profits – they increase or decrease together.

When broker B_1 downgrades the lower-level service obtained from its provider S_1 (*i.e.*, $q_1 < Q_1$), we see from the brokers' and providers' price plots (Figure 2) that all brokers and providers can offer their service at higher prices and make more profit compared to the case when B_1 upgrades the obtained service from S_1 (Figure 3). Similarly, by comparing the behavior for higher values of q_2, where $q_2 > Q_2$, with that for lower values where $q_2 < Q_2$, we observe that a better strategy for broker B_2 is to upgrade the lower-level service that it obtains from S_2 (*i.e.*, $q_2 > Q_2$). This happens because upgrading q_2 or downgrading q_1 leads to a larger gap between q_1 and q_2, therefore the two sets of broker and provider can offer more differentiated services at higher prices. In fact, this follows the maximum differentiation principle.

In this setting, since we have the least number of brokers to compete, it is more likely that monopoly situations arise. For example, for $q_1 = 29$ (Figure 3), the market exhibits abnormal behavior when the gap between q_1 and q_2 is small, while the gap between providers' qualities and brokers' qualities is large. Specifically, the market approaches a monopoly where B_2 has a small market share when q_2 is closer to the service quality of S_1 (Q_1). Observing the results when the values of q_2 are close to 30, we note that, although the providers' game is a monopoly at some points (where S_2's price $r_2 = 0$), the brokers' game is not, and B_2 can have a small share of the market D_2 while it gets service from provider S_1. This is because when the gap between q_1 and q_2 is not significant, most of the customers prefer the cheaper service provided by broker B_1. When the market is a monopoly, the provider or broker who remains in the market can increase its price while ensuring that the other competitor cannot enter the market even if that competitor lowers its price to equal its cost, thus there is no way for the competitor to make profit and is prevented from entering the market.

On the other hand, for $q_1 = 29$, when broker B_2 is upgrading the service quality obtained from S_2, *i.e.*, $q_2 > Q_2$, as the gap between q_2 and Q_2 gets larger, S_2 starts to decrease its price to cover the cost of the quality upgrade for B_2 so as not to lose its market share. Since the value of q_1 is somewhere between Q_1 and Q_2, it is more economical for B_1 to buy service from S_2 rather than S_1 at the optimal prices, *i.e.*, the optimal price of S_1 violates constraints (11) and it should update its price r_1 as we explained in Section 3.6. Consequently, S_2 should also update its price. Since there is a substantial gap between q_1 and q_2, both providers can compete in the market.

4.2. Positive Utility Results

Observation 3. *In the positive utility game, increase in the profit of one player is at the expense of the other player.*

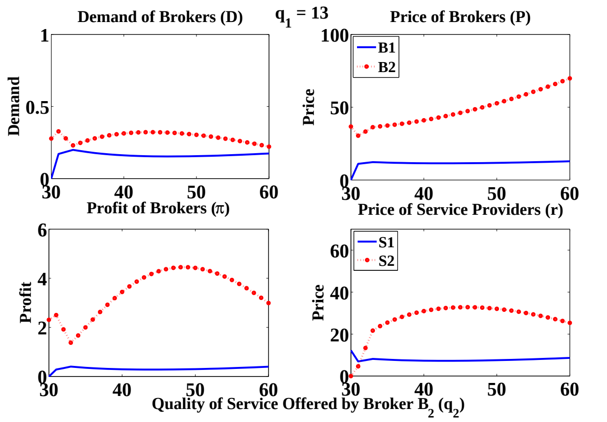

Figure 4: Price, profit and demand distribution for brokers, and price of providers, for the positive utility game, B_1 downgrading the quality $Q_1 = 20$, $q_1 = 13$, $Q_2 = 45$, $30 \leq q_2 \leq 60$, as the service quality offered by broker B_2 changes.

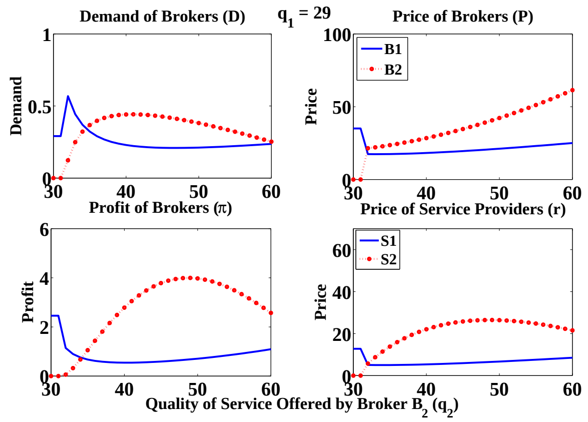

Figure 5: Price, profit and demand distribution for brokers, and price of providers, for the positive utility game, B_1 upgrading the quality $Q_1 = 20$, $q_1 = 13$, $Q_2 = 45$, $30 \leq q_2 \leq 60$, as the service quality offered by broker B_2 changes.

We now consider the case of positive utility competition. Intuitively, we expect to see some restriction on the prices for all brokers and providers, otherwise they lose part of the market for which the utility $(\theta q - p)$ is negative. Therefore, it is a compromise between price and demand. The numerical results confirm this intuition. Comparing the prices of brokers and providers under positive utility and unconstrained utility, for the same conditions, shows that *the highest prices under positive utility are below half of the prices in the latter case, while the demands are less as well*; compare plots in Figures 2 and 3 with plots in Figures 4 and 5.

Also, in this positive utility game, whether brokers upgrade or downgrade the service obtained from their providers, the behavior is different from that in the unconstrained utility game. Specifically, since the positive utility market is more sensitive to prices, a smaller gap between the service quality offered by the broker and the quality it gets from its provider yields more profit. Furthermore, while for both brokers, slightly upgrading the service obtained from lower-level providers (and in turn, selling a higher quality service to customers) is generally more profitable (compare profit plots in Figure 4 and Figure 5), B_2 gains more profit from a larger quality gap caused by lower q_1.

Unlike the unconstrained utility game, if profit increases for one player, profit decreases for the other player. Another interesting observation from these plots arises when there is a monopoly in the market: while there are conditions under which broker B_1 can lose its market share ($D_1 = 0$ when $q_2 = 30$ in Figure 4), service provider S_1 can manage to stay in the market under all conditions.

4.3. Sensitivity to Quality-Conversion Cost

In our model, we assume that brokers can change the quality of service that they buy from the service providers so they offer a new service that meets the requirements of customers. Modeling the real cost function for converting the service quality is complicated and our economic model clearly does not capture the complex structure of the market. For the sake of analytical tractability, we have chosen a quadratic function $c(q_i - Q_i)^2$, that intuitively captures the reasonable assumption that the cost of service quality upgrade/downgrade by a broker increases more rapidly as (the difference in) service quality increases.[3] To study the sensitivity of our results to this assumption, we have analyzed the effect of this quality conversion cost by examining different values for c. For relatively small values of c, brokers are able to change the quality of the obtained (lower-level) services as much as they want to achieve more service differentiation from other brokers. As the value of c gets larger, the cost of con-

[3]Consider, for example, the service offered by a Content Distribution Network (CDN) provider who manages the degree of replicating content to meet a certain delivery delay requirement. In this case, the cost could be modeled as a function of the area over which the content is replicated, *i.e.*, the cost is proportional to the square of the radius/distance, where a larger distance reflects higher content replication and thus lower delivery delay (higher/better quality of service).

verting the lower-level quality increases, and consequently there is an optimal point for changing that quality as a broker maximizes service quality differentiation from other brokers. Specifically, while for a broker, picking a quality beyond that (optimal) point decreases the profit of that broker – because of the high cost of converting the lower-level quality that it is getting – the profit of the other broker(s) still increases because of maximum service quality differentiation. In our numerical analysis, we assume that the service qualities which B_1 and B_m pick, are not beyond the optimal quality (that maximizes their profit).

4.4. Results with Three and Four Brokers

Observation 4. *When there are more competitors in the market, the gap between their service qualities decreases, the competition on the price becomes tougher and brokers should offer their services at lower prices to be able to attract customers and make profit.*

Observation 5. *In the market with more than two brokers, though the maximum differentiation between the service qualities of brokers reduces the intensity of competition, the cost that brokers undergo is also playing an important role. There are situations where violating the maximum differentiation rule in order to buy service from the other provider gives rise to higher broker's profit.*

In this section we extend our setting to three and four brokers to see if the maximum differentiation principle holds for more brokers. We assume that two brokers offering the lowest and the highest quality of service to users are already in the market and define the range of feasible quality. We then let the other one or two brokers enter the market with a quality level chosen in such range. After fixing a quality level, the third (and fourth) broker obtains service from the (lower-level) provider that minimizes the quality difference between them. This in turn minimizes the broker's cost in providing service to its customers. As in previous case studies with only two brokers, we show results at the equilibrium of the game by identifying indifferent customers between available service qualities. We also apply all constraints on the providers' level to have an oligopoly market.

4.4.1. Results Considering Three Brokers

We consider the game with $\theta_{min} = 1,\ \theta_{max} = 70$, two providers S_1 and S_2 with $Q_1 = 30$ and $Q_2 = 60$, and three brokers, B_1, B_2 and B_3, with qualities q_1, q_2 and q_3, respectively. We assume that the quality levels of B_1 and B_3 are fixed and we let the quality of broker B_2 change in the interval (q_1, q_3). Broker B_2 chooses the service provider with least quality difference to reduce its (service conversion) cost. Given the above settings, we observe a tipping point for the quality of broker B_2 (q_2): for $q_2 < 45$, B_2 chooses provider S_1, and for $q_2 > 45$, B_2 chooses provider S_2; for the frontier value of $q_2 = 45$, although there is no quality differentiation between the two (lower-level) providers, we observe that downgrading the service has less cost than upgrading it, therefore B_2 chooses to get its service from S_2. The jump in profit at $q_2 = 45$ in Figure 6 is because of B_2's switching provider.

As we can see in Figure 6, for each of brokers B_1 and B_3, which have been already in the market, it is more profitable if broker B_2 chooses to offer a quality with the maximum difference from their quality, while for broker B_2 it is more profitable to have maximum difference with both B_1 and B_3. As we have observed in the case of two brokers, it is not advisable to choose a quality of service similar to that of other providers. Intuitively, this is because the more difference in the service quality that they offer customers, brokers are more likely to serve customers at a higher price. We note this by observing that the optimal quality for broker B_2 is the average of the other fixed brokers' qualities (q_1 and q_3). For example, in the first plot from left in Figure 6, the optimal $q_2 = 50$, which is obtained from $(q_1 + q_3)/2 = (10 + 90)/2$.

We also change the fixed service qualities of brokers B_1 and B_3 toward the optimal quality for B_2 to see how the market changes and compare such results with those of Hotelling's location model with more than two firms [71]. As we can see in Figure 6, unlike the Hotelling's model [71] where corner firms have a tendency to move toward internal firms, here all brokers make less profit when quality differentiation decreases. Although, for broker B_3, its market share increases (Figure 7), the effect of dropping the price is more pronounced than the extra share of the market and leads to lower profit.

Main Result: This means that, *unlike the Hotelling's location model, for three firms, the market follows the maximum differentiation principle and brokers make more profit when their service qualities are more different from each other.* In the following setting we study four brokers to see if this pattern repeats.

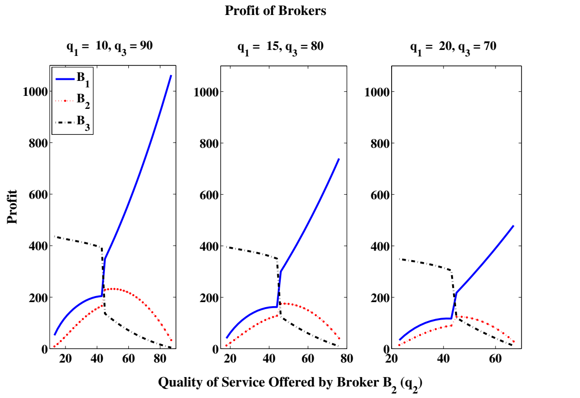

Figure 6: Profit of each broker in a three-broker case varying q_2.

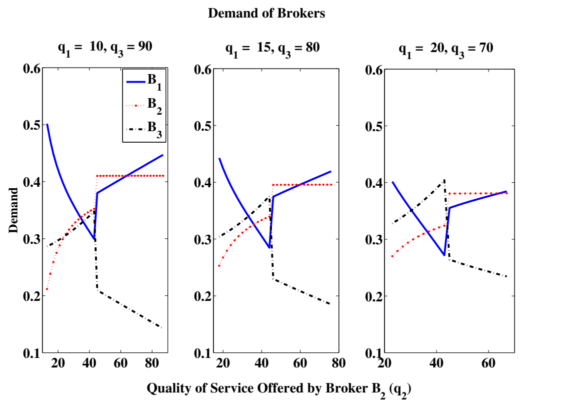

Figure 7: Demand of each broker in a three-broker case varying q_2.

4.4.2. Four Brokers

In this setting, we consider a scenario with two brokers, B_1 and B_4 already in the market and offering fixed service qualities $q_1 = 10$ and $q_4 = 90$, respectively, and two other brokers, B_2 and B_3, that enter the market later. Without loss of generality, we assume that $q_2 < q_3$. Figure 8 shows the changes in profit for brokers B_2 and B_3. We omit the results for B_1 and B_4 since they follow the same pattern as in the previous case study with three brokers, *i.e.*, the more differentiation between their qualities and those we set for B_2 and B_3, the higher is their profit. This means that such brokers are not the decision makers in this situation.

As we observe in Figure 8, for broker B_3, whose quality is between q_2 and q_4, the optimal quality q_3 value is one that yields maximum differentiation from both qualities q_2 and q_4, which is close to the average of q_2 and q_4. For broker B_2 we expect instead that the optimal quality level is around $q_2 = 37$, that is, the quality with maximum difference from q_1 (10) and the optimal q_3 (which equals 64 given maximum quality differentiation among all brokers). However, we observe that the optimal quality for B_2 is at $q_2 = 45$, when broker B_2 switches from provider S_1 to provider S_2 and instead of upgrading the quality, downgrades the service that it obtains from provider S_2 (recall that $Q_1 = 30$ and $Q_2 = 60$). To understand why B_2 violates the maximum differentiation rule, we analyze the situations under both $q_2 = 37$ and $q_2 = 45$.

For $q_2 = 37$, the observed optimal value for B_3 is $q_3 = 56$, and not the expected value of $q_3 = 64$. To explain this situation, we should consider that in making profit, besides quality differentiation with other competitors (brokers), the cost of buying the lower-level service is also important. In this case, broker B_3 makes more profit if it chooses $q_3 = 56$ and downgrades the service it obtains from S_2 (recall $Q_2 = 60$) instead of choosing $q_3 = 64$ and upgrading the service. Broker B_3 can then offer a quality-price combination that attracts more customers, while because of the sufficient gap between q_2 and q_3, the competition on the price is not tough. However, in this situation, broker B_2 is upgrading the service that it obtains from provider S_1 (recall $Q_1 = 30$) and to compete with broker B_3, it cannot offer a high price, and the profit that it makes is relatively low.

On the other hand, for $q_2 = 45$, the situation is reversed. B_2 downgrades the service that it obtains from provider S_2, while B_3 at its optimal point is upgrading the service. So the combination of quality-price of broker B_2 attracts more customers which leads to making more profit. Therefore in this game, besides maximum quality differentiation, the cost that brokers undergo is also playing an important role and sometimes brokers should compromise on maximum differentiation to reduce their cost and make more profit.

Assuming rational players, *i.e.*, the two new brokers pick the quality that maximizes their profit, we compare the price of the service that such brokers offer for the case studies of three and four brokers. In the case of three brokers, we observe that the optimal quality for broker B_2 is at $q_2 = 50$ while $q_1 = 10$ and $q_3 = 90$. The optimal price for brokers in this setting is $p_1 = 2092$, $p_2 = 3203$ and

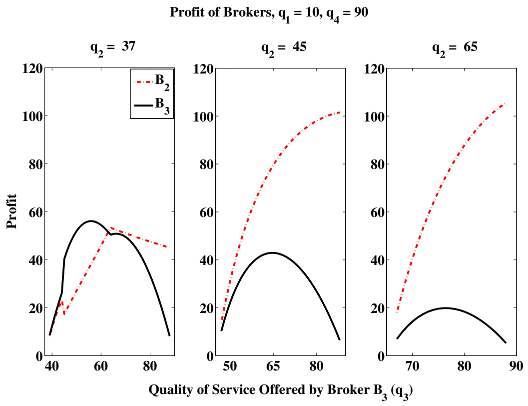

Figure 8: Profit of brokers B_2 and B_3 in a four-broker setting.

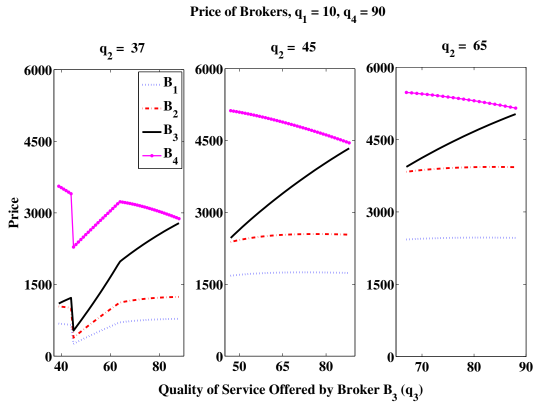

Figure 9: Price of each broker in a four-broker setting.

$p_3 = 5447$, respectively. When four brokers are playing the game, the optimal quality for broker B_2 is $q_2 = 45$ and for broker B_3 is $q_3 = 66$. In this situation, the optimal prices are $p_1 = 1746$, $p_2 = 2535$, $p_3 = 3442$ and $p_4 = 4869$. As we can see, the price of service with quality 10 and 90 drops from 2092 and 5447 to 1746 and 4869, respectively.

4.5. Generalized Market and Main Results

In previous sections we considered markets with up to four brokers and analyzed them. However, modeling a real market with more brokers is very complex. While Equations (3)–(12) can be used to derive all demands, prices, and profits so as to analyze a market with any number of brokers, as the number of variables (brokers) increases, the complexity of finding a closed-form solution for each element of the market increases[4]. However, we describe here our main results for a generalized scenario:

- If Δq $(q_{max} - q_{min})$ increases, there is more room for brokers to differentiate their quality, and therefore, prices can be higher; this in turn leads to higher profit.

- In general, the maximum differentiation principle applies to all players, *i.e.*, players make more profit as the difference between their quality and rivals' qualities is higher.

- An increase in the number of brokers leads to a lower differentiation between the service quality offered by brokers; this means that prices are lower compared to the situation in which the market has a lower number of brokers.

- When the competition level increases (*i.e.*, larger number of brokers, limited quality range, *etc.*), the cost of converting quality obtained from lower-level service providers plays an important role than what the maximum differentiation principle dictates. This is true for a broker whose offered service deviates almost equally from any of the service providers' quality, *i.e.*, $|q_i - Q_1| \approx |q_i - Q_2|$. In this situation, violating the maximum differentiation principle in one direction, to get the service that yields less cost, leads to more profit for the broker, though at the expense of rival's profit (because of a lower quality differentiation).

5. Cooperative Game

In Section 3, we studied the pricing strategy in a two-layered network market, where service providers and brokers compete at different levels in an oligopoly market to maximize their profit. We modeled a non-cooperative game, in which all players try to optimize their own profit independently. The non-cooperative nature of players might have a negative impact on the profit of players and leads them to less profit in total. For example, in Figure 6, while the optimal profit for the new broker (B_2) is to pick a quality at the middle of existing qualities, the total profit of brokers B_1 and B_2 is

[4] All code related to analytical and numerical solutions can be found at `https://github.com/MaryGhasemi/Multi-Layer-Market`

maximized when B_2 picks a quality close to that of B_3.

In this section, we consider a partially cooperative game. In our setting, there are two service providers with different qualities at the lower level and two or more brokers on top of them. When a new broker enters the market, it chooses the quality that maximizes its profit; however, it happens that the quality chosen by the new broker is not the best for brokers that are already in the market. Here, we study the oligopoly market at the brokers' level, with two or more brokers, where a new broker enters the market and cooperates with one of the existing brokers rather than competes. We study the impact of this cooperation on the quality that the new broker chooses, the prices set by players in the market (brokers and service providers), and also customers' social welfare. While in most situations, cooperation helps brokers make more profit, albeit with a negative impact on customers' utility, there are cases where both coalition brokers and customers benefit from the cooperation.

5.1. Profit Sharing Policy

We study a market with two service providers, and two or more brokers, denoted by S_i's and B_i's, respectively, where a new broker enters the market. Service providers and brokers compete in two different levels in a Bertrand competition to find their best strategy. We consider the market under different situations where the new broker cooperates with one of the other brokers. We assume that the new broker picks a quality in the range of available quality in the market. We define the demand function as Equation (2):

$$D_i(p_1, p_2, ..., p_m) = \frac{\theta_i^* - \theta_{i-1}^*}{\Delta\theta}$$

where $\theta_i^* = \frac{p_{i+1}-p_i}{q_{i+1}-q_i}$.

The profit function for broker B_i who buys services from service provider S_k is also given as Equation (6). In this cooperative game, when two brokers cooperate, we assume that they maximize the summation of their profits, *i.e.*, $\Pi_{i+j} = \Pi_i + \Pi_j$.

5.1.1. Sharing proportional to demand

One way of sharing the profit between cooperative brokers is to divide it proportional to the demand that each broker supports, *i.e.*, $\Pi_i = \Pi_{i+j} \times \frac{D_i}{D_i+D_j}$.

Though this strategy seems to be fair, it leads to more competition and less profit. This is because, in this setting, each of cooperating brokers wants to have more share of the market (demand) to gain more profit. Although the cooperating brokers want to maximize their total profit, since each of them wants to maximize its own revenue as well, this leads to a more competitive market and much lower prices compared to the non-cooperative market. In the end, at steady state, the profit that each broker gains is less than that under the non-cooperative game. Therefore, it is not rational for brokers to cooperate under this policy.

5.1.2. Sharing proportional to profit at optimal point of non-cooperative game

The other way of splitting the profit between cooperating brokers is to share it proportional to their profit at the optimal solution of the non-cooperative game. This policy also gives more incentive to the broker that gains more from cooperation. So, for cooperating broker B_i, the profit is calculated by:

$$\Pi_i = \Pi_{i+j} \times \frac{\Pi'_i}{\Pi'_i + \Pi'_j}$$

where Π'_i and Π'_j are non-cooperation profits at equilibrium. To find the optimal price, like the non-cooperative game, every broker B_i solves $\partial\Pi_i/\partial p_i = 0$. The game in the second stage, between service providers, is again a Bertrand competition and follows the same settings as we discussed in Section 3.

5.2. Experimental Results

We consider two different settings with 3 brokers and 4 brokers. In both settings, the first and last brokers are in the market with the lowest and highest quality of service, respectively, and the other brokers enter the market with a quality between them. We study the effect of cooperation between different brokers on brokers' utility as well as users' utility. In the following subsections, we start with our main observations followed by a detailed analysis of our results.

5.2.1. Three Brokers

We consider a setting with 3 brokers, where B_1 and B_3 are in the market with lowest and highest quality of services, which in this case are $q_1 = 10$ and $q_3 = 120$, and B_2 is entering the market.

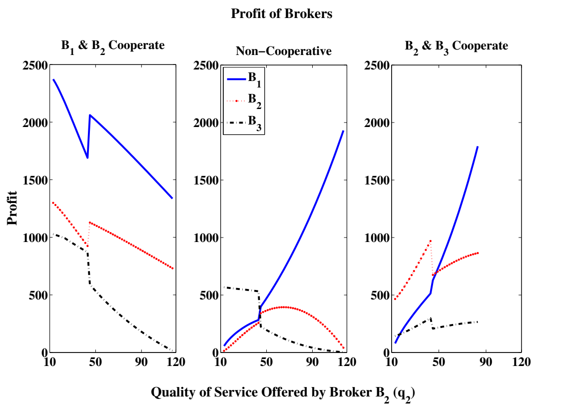

Figure 10: Profit of brokers in cooperative & non-coperative cases, for varying q_2.

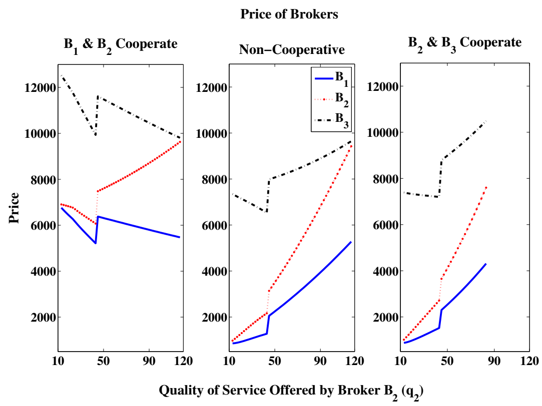

Figure 11: Price of brokers in cooperative & non-coperative cases, for varying q_2.

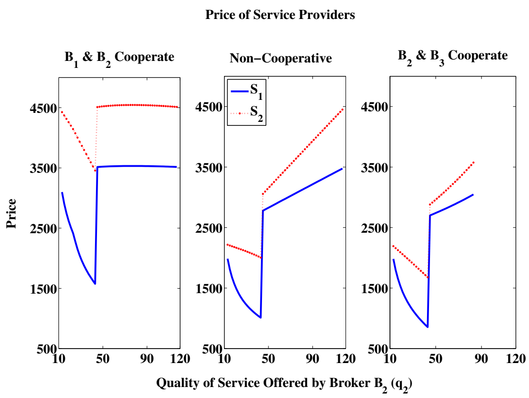

Figure 12: Price of service providers in cooperative & non-cooperative cases, for varying q_2.

The demand function is defined like in the non-cooperative game, as well as the profit function for providers and brokers. Assuming B_2 picks a quality that maximizes its profit ($q_2 = 65$; see Figure 10), the profit of brokers in the non-cooperative game at equilibrium is $\Pi_1 = 720$, $\Pi_2 = 394$ and $\Pi_3 = 121$. We assume that the cooperating brokers share their total profit proportional to these profits. There are two different cooperation scenarios for broker B_2: one is cooperation with B_1, and the other is cooperation with B_3. We analyze both cases to see how prices and profits change for brokers and providers.

5.2.1.1 B_1 and B_2 Cooperate

Observation 6. *If broker B_2 picks a quality close to B_1, all players can offer their services at higher prices and make more profits.*

Observation 7. *While all players in the market benefit from the cooperation of B_1 and B_2, customers pay much higher prices for the same or lower quality services, compared to the non-cooperative game.*

When B_1 and B_2 cooperate, if B_2 chooses quality q_2 closer to q_1, they can offer their services at higher prices and make a significantly larger profit compared to the non-cooperative game. In this case, other players, including B_3, S_1 and S_2, can also offer their services at higher prices, therefore the market is equilibrated at higher prices. If broker B_2 picks a service with higher quality, *i.e.*, gets closer to the quality of broker B_3, the competition between B_2 and B_3 gets more serious. Consequently, B_2 should set its price in a lower range, and so other players should do the same. The closer q_2 gets to q_3, the competition gets more tense and the prices get closer to the prices in the non-cooperative game. The left plots in Figures 10 and 11 present the profit and price of brokers, and Figure 12 shows the price of providers, when B_1 and B_2 cooperate. The plots in the center illustrate the non-cooperative competition. As we observe in Figure 10, the optimal point for broker B_2 is at quality $q_2 = 13$ where its profit is maximized. As Figure 13 shows, since broker B_1 has the lowest quality and price, it does not lose its share of the market.

5.2.1.2 B_2 and B_3 Cooperate

Observation 8. *B_2 cannot pick a quality close to B_3, nor they can set their prices as high as the prices in the B_1-B_2 cooperation.*

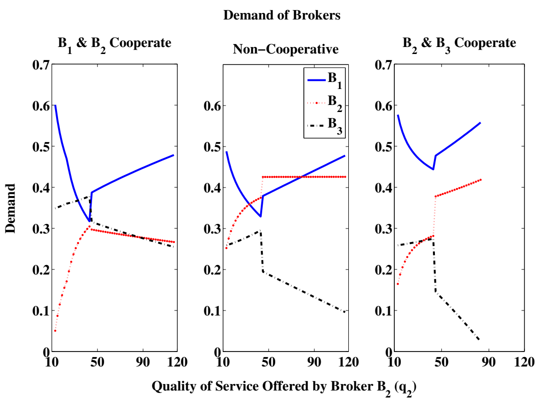

Figure 13: Demand of brokers in cooperative & non-cooperative cases, for varying q_2.

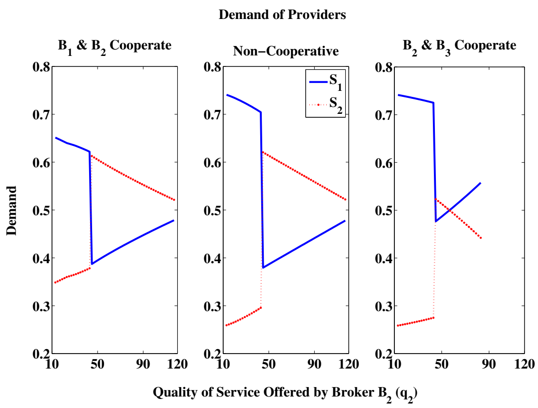

Figure 14: Demand of providers in cooperative & non-cooperative cases, for varying q_2.

Observation 9. *At the optimal point for B_2 and B_3, while they make more profit, they also offer their services at lower prices, compared to the equilibrium point of the non-cooperative market. The profit comes mostly from reducing the cost of obtaining the service from lower-level providers.*

In the case of collaboration between B_2 and B_3, unlike the collaboration of B_1 and B_2, if B_2 picks a quality close to q_3, they cannot set high prices, otherwise no one would prefer to buy from B_3 and broker B_3 is out of market. Therefore, the prices are close to those of the non-cooperative market. On the other hand, if broker B_2 chooses a lower quality with fair difference from q_3, it gets into competition with broker B_1. Therefore, in this collaboration game, the prices cannot be set too high, because either it causes broker B_3 to get out of the market, or B_2 and B_1 get into competition to increase their share of the market. However, in this setting, the price of service providers is lower than that of the non-cooperative game. This is because the prices chosen by B_2 and B_3 lead them to less demand, and consequently less demand for S_2 as well (right side plots in Figures 13 and 14). This situation makes S_2 lower its price to attract more demand, which makes S_1 pick a lower price as well. The best strategy for B_2 and B_3 is to buy from different service providers, so force them into more competition. Also, the optimal quality for broker B_2 is at the highest quality in which buying from S_1 still has less cost than buying from S_2. At this quality, the providers' prices equilibrate at the lowest range and brokers can benefit from that.

Table 1 presents the prices and profits of brokers and service providers at the quality of q_2 in which the profit of broker B_2 is optimized, in different situations, *i.e.*, in the non-cooperative case and in the case of cooperation of B_2 with B_1 or B_3. It also shows the percentages of change compared to the non-cooperative values. For the price and profit of broker B_2, since the optimal quality q_2 changes in different situations, the shown percentages are based on changes per unit of quality. As we observe it Table 1, every player benefits from cooperation of B_1 and B_2 by setting a higher price. However, in the case of collaboration between B_2 and B_3, only B_2, B_3 and S_1 have a higher profit compared to the non-cooperative case, but it is better economically for customers as prices are lower.

	Non-Coop	B_1 & B_2 Cooperate		B_2 & B_3 Cooperate	
q_2*	65	13		43	
p_1	2840	6761	138%	1522	−46.5%
p_2	4664	6907	640%	2711	−13%
p_3	8338	12520	50%	7199	−14%
Π_1	720	2373	229%	512	−29%
Π_2	394	1299	229%	969	145%
Π_3	121	1021	743%	286	136%
r_1	2974	3099	4%	856	−72%
r_2	3446	4428	28%	1678	−52%
U_1	403	688	69%	472	16%
U_2	2737	3082	12%	919	−66%

Table 1: Prices and profits of brokers and service providers at optimal quality q_2 in different scenarios and percentages of change compared to the non-cooperative case.

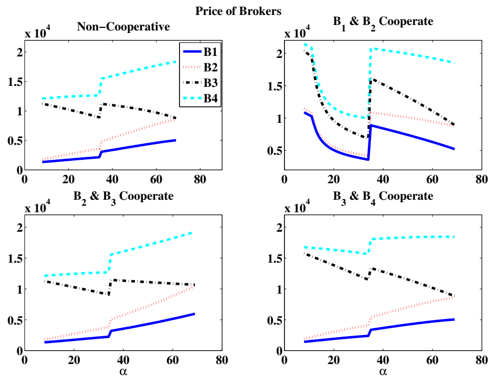

Figure 15: Price of brokers in cooperative & non-cooperative cases, for varying α.

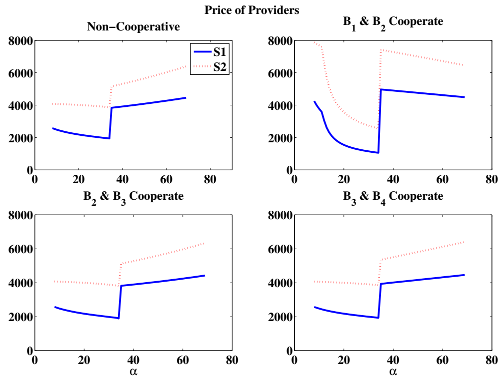

Figure 16: Price of providers in cooperative & non-cooperative cases, for varying α.

5.2.2. Four Brokers

In this setting, we have four brokers along with two service providers S_1 and S_2 with $Q_1 = 30$ and $Q_2 = 60$. Brokers B_1 and B_4 are in the market with the lowest and highest quality of $q_1 = 10$ and $q_4 = 150$, and B_2 and B_3 enter the market choosing a quality between q_1 and q_4. Without loss of generality, we assume that $q_2 < q_3$. To reduce the complexity of varying q_2 and q_3, we introduce a new variable α and set $q_2 = q_1 + \alpha$ and $q_3 = q_4 - \alpha$. We vary α from 8 to $\frac{q_4 - q_1}{2}$; α is used to control the quality gap between q_1 and q_2, and q_3 and q_4.[5] Then, we monitor the changes in the market as B_2 and B_3 change their qualities away from the quality endpoints (q_1 and q_4) and toward the center, where they get closer to each other. Since there are two brokers changing their qualities, we do not have any single optimal point in the four-brokers game, as we have in the three-brokers game, so we compare the result of cooperation with that of the non-cooperative game in the same setting.

We consider the market in different situations where there is no cooperation, or there is cooperation between B_1 and B_2, B_2 and B_3, or B_3 and B_4.

5.2.2.1 B_1 and B_2 cooperate

Observation 10. *When α is small,* i.e., *q_1 and q_2 are close to each other, B_1 and B_2 can offer their services at higher prices and make more profit.*

Observation 11. *When the structure of the brokers' market imposes a tense competition in the service providers' market, brokers and customers benefit from this competition; while brokers make profit as a result of decrease in the cost of buying services, customers buy services from brokers at lower prices.*

In this setting, when the qualities of service of B_1 and B_2 are close to each other, they can set a high price for their services. Other brokers also raise their prices. As B_2 increases its quality q_2, and q_2 and q_3 get closer to each other, the price of brokers drops. But the drop in prices of B_3 and B_4 is more than that of B_1 and B_2's. Indeed, B_1 and B_2 make profit by having higher prices, compared to the non-cooperative game, while prices of B_3 and B_4 are even lower than their prices in the non-cooperative game (Figure 15 up right plot); B_3 and B_4 make more profit by attracting more demand, which is also the case for service provider S_2. As the price of S_2 decreases and B_2 picks a higher quality, the competition between S_1 and S_2 gets more intense and they decrease their prices (Figure 16). This situation holds until B_2 switches from S_1 to S_2. Figure 17 shows the profit of brokers in the non-cooperative game and under this B_1-B_2 cooperation.

5.2.2.2 B_2 and B_3 cooperate

Observation 12. *While quality differentiation between competitors is not large, the effect of competition outweighs the effect of cooperation. This is the scenario when $\Delta q_{coop} \gg \Delta q_{comp}$, where Δq_{coop} is the quality differentiation between cooperative brokers and Δq_{comp} is the quality differenti-*

[5] If q_2 gets too close to q_1, or q_3 gets too close to q_4, B_1 or B_4 are out of the market. Thus, α starts from a value so everyone can compete in the market.

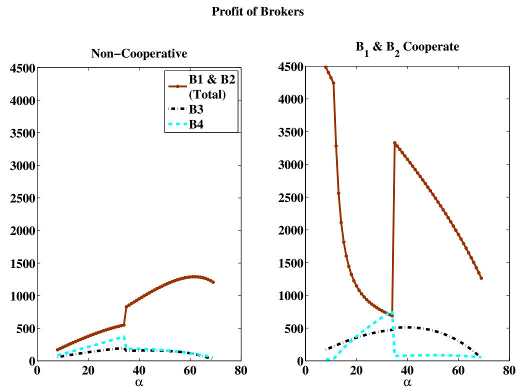

Figure 17: Profit of brokers in non-cooperative game and B_1-B_2 cooperation game, for varying α.

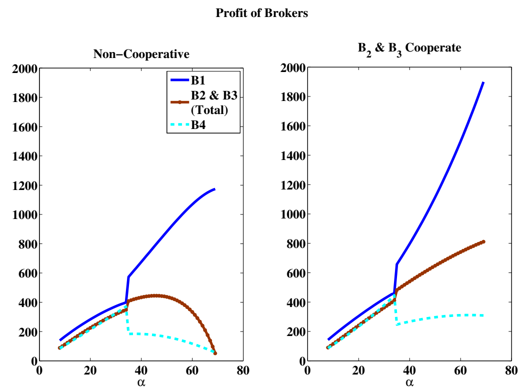

Figure 18: Profit of brokers in non-cooperative game and B_2-B_3 cooperation game, for varying α.

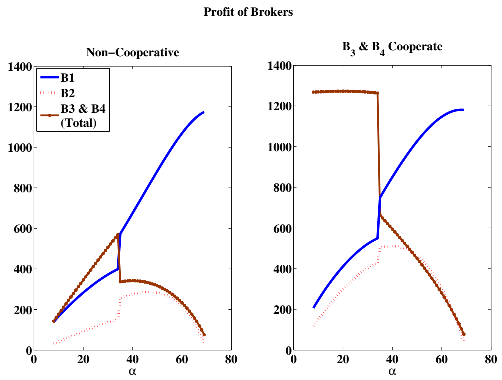

Figure 19: Profit of brokers in non-cooperative game and B_3-B_4 cooperation game, for varying α.

ation between cooperative brokers and other neighboring brokers.

In the case of cooperation of B_2 and B_3, when α is small, *i.e.* q_2 is close to q_1 and q_3 is close to q_4, B_2 and B_3 are in high competition with B_1 and B_4, respectively; therefore, their cooperation have almost no effect on the system and the prices are almost the same as the non-cooperative game (Figure 15 down left plot). As α gets bigger, *i.e.* q_2 and q_3 get closer to each other, the competition with their rivals is less intense and they can set their price to a higher value and the market is equilibrated at higher prices. Figure 18 compares the brokers' profit in the non-cooperative game and under this B_2-B_3 cooperation setting.

5.2.2.3 B_3 and B_4 cooperate

Observation 13. *When B_3 and B_4 cooperate, as α increases, as long as B_3 is not in competition with B_2, their total profit remains high.*

In this configuration, when α is small, *i.e.* q_3 and q_4 are close to each other, B_3 and B_4 can set their price to higher values, but their prices are not as high as B_1 and B_2 set in their cooperation (compared to the non-cooperative case); otherwise B_4 is out of the market. As α increases, unlike the other cooperations, the total profit of B_3 and B_4 remains high. This is because when α increases, the quality differentiation between B_1 and B_2 increases and they can then increase their prices. Meanwhile, the quality differentiation between B_3 and B_4 also increases and they can attract more share of the market. When q_3 gets closer to q_2 and there is more competition between B_2 and B_3, the prices and profits get closer to those in the non-cooperative game. Figure 19 shows the brokers' profit in the non-cooperative game and the B_3-B_4 cooperation game.

5.2.2.4 Users' Utility

Observation 14. *Customers' welfare is higher when competition is tough.*

Observation 15. *If brokers can impose more competition on the lower-level providers, both brokers and customers benefit from that competition.*

To compare the customers' welfare in different cases, we calculate the summation of users' utility $(\theta q_i - p_i)$ and normalize it by dividing by maximum

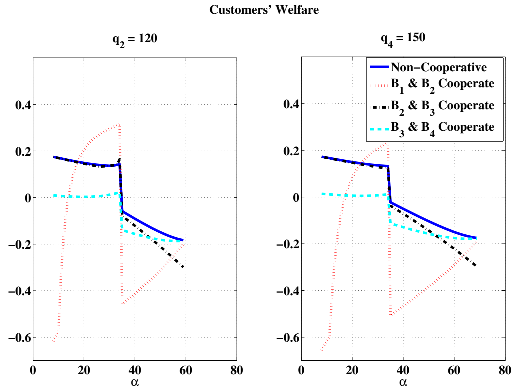

Figure 20: Normalized total customers' welfare in non-cooperative & cooperative games, for varying α.

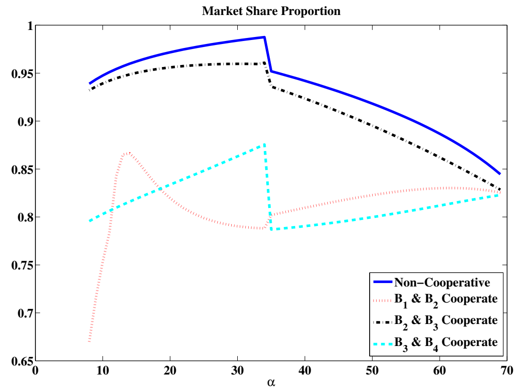

Figure 21: Fairness measure of brokers' market share in non-cooperative & cooperative games, for varying α.

total utility, which is obtained when all customers buy service from the highest quality broker (B_4 in this case) at zero price. Also, we define a *fairness metric* as $\frac{(\sum D_i)^2}{n \times \sum D_i^2}$, where D_i is the demand of broker i and n is the number of brokers. This metric shows us how evenly the market share is distributed among brokers. Specifically, this fairness metric approaches 1 when demands are equal, and approaches zero otherwise [74]. Figure 20 presents the customers' welfare in different games with or without cooperation, for two different markets with different quality ranges, where in the left plot $q_4 = 120$ and in the right plot $q_4 = 150$. For the non-cooperative game, when α is small, the competition of B_1-B_2 and B_3-B_4 is intense and their prices are low, so users benefit from this competition. The same story is true for B_2-B_3 cooperation, when α is small; however, when q_2 and q_3 get close to each other, the users' welfare is less than that in other games. As we can see in Figure 20, B_1-B_2 cooperation game has the lowest users' welfare, except for some values of α, when B_2 buys service from S_1, and there is intensive competition between S_1 and S_2. In this situation, as we explained above (for B_1-B_2 cooperation), S_2 and consequently B_3 and B_4 make more profit by attracting more market demand instead of by setting higher prices, therefore the users' welfare can be higher than other cases.

Figure 21 shows the fairness measure of market share for brokers. It is clear that when there is no cooperation and brokers have a fair difference between their service qualities, and also there is competition between service providers (B_1 and B_2 buy from S_1, and B_3 and B_4 buy from S_2), the market is almost evenly shared among brokers.

5.3. General Results for Partial Cooperative Game

The complexity of the partial cooperative market is the same as we would have in a competitive market. While Equations (3)–(12) work for any number of brokers, the closed-form solutions are complex. However, we describe some general results that hold for a partial cooperative market with a number of brokers greater than two:

- In a partial cooperative market, all brokers benefit from the ongoing cooperation; however, customers pay much higher prices for the same or lower quality of service, compared to the non-cooperative game.

- The cooperation of B_1 (the broker with the lowest quality) with B_2 (the broker with the next quality level) yields the highest profit to all players, compared to cooperation between other brokers. This happens because B_1 and B_2 can increase their price much more than other players in cooperation without getting out of the market; therefore other players also can set a higher price.

- If the quality differentiation between competitors is not large, cooperation has little effect on increasing profit for (cooperating) brokers.

6. Conclusion

In this paper, we developed a game-theoretic model that captures the interaction among players in a multi-level market. In our model, brokers, as the intermediaries between users and service providers, adapt the quality of the service that they get from lower-level providers so as to attract more customers and maximize their profit. The game consists of two service providers, two, three or four brokers, and users, though we study more extensively the case with two brokers. Numerical results show that the more differentiation between the quality of service offered by brokers, the higher is their profit. However when the competition is intense, besides quality differentiation, cost plays an important role and forces brokers to compromise on quality differentiation with their competitors to reduce cost and make more profit. An interesting result in the two brokers game is that although players compete for more profit, the competition only affects their market share; the profit increases for one player if it increases for the other one. But this is not the case for more brokers. When there are more than two brokers, the market is more competitive and brokers should offer their services at lower prices to be able to stay in the market.

We also considered a partial cooperative game where incoming brokers decide to cooperate with one of the brokers in the market; in this game, cooperative players maximize their total profit instead of their own profit. The numerical results show that in most of the cases, all players benefit from the cooperation of a subset of players; indeed, cooperation means less competition. However, the benefit from cooperation depends on the quality differentiation between cooperating brokers and also the quality differentiation of cooperating brokers with other brokers they compete with. The highest profit occurs when the service qualities of coopering brokers are close to each other, with a substantial quality difference from other brokers. Also, players make more profit when the incoming broker cooperates with the broker with lowest quality rather than cooperates with the broker with highest quality; in cooperation with the high quality broker, if they set their price too high, the high quality broker loses its market share and goes out of the market. Furthermore, when the service quality offered by brokers is distributed almost uniformly, the cooperation does not have much impact on the market.

Although in most cooperative settings, customers do not benefit from brokers' cooperation, there are situations where cooperation can yield lower prices. In these cases, the combination of brokers' service quality and their market shares impose a high competition on the service providers' level, and as a result of such competition, they lower their price. Consequently, the cost of providing services is reduced for brokers and they can make more profit by incurring less cost.

We believe our model and findings can guide the design and analysis of current and emerging brokered (service-oriented) systems, including Software Defined eXchanges (SDX) stitching (virtual or physical) resources from multiple domains to offer a range of software defined services, *e.g.*, a video marketplace.

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Appendix: Analytical Solution for the case of Two-Brokers, Two-Service Providers

In this appendix, we show the detailed derivation of our analytical solution for the particular case of two service providers S_1 and S_2, and two brokers B_1 and B_2. As discussed in Section 3, we assume that customers have different quality preferences, modeled by $\theta q - p$, where θ is the customer's marginal willingness to pay for quality q, and p is the price of service. Under these assumptions, the indifferent customers, θ^* satisfy:

$$\theta^* q_1 - p_1 = \theta^* q_2 - p_2 \Leftrightarrow \theta^* = \frac{p_2 - p_1}{q_2 - q_1}.$$

If we assume a uniformly distributed θ, the demand for each broker, B_1 and B_2, is given by:

$$D_1(p_1, p_2) = \frac{\theta^* - \theta_{min}}{\Delta\theta} = \frac{1}{\Delta\theta}\left(\frac{p_2 - p_1}{q_2 - q_1} - \theta_{min}\right)$$
$$D_2(p_1, p_2) = \frac{\theta_{max} - \theta^*}{\Delta\theta} = \frac{1}{\Delta\theta}\left(\theta_{max} - \frac{p_2 - p_1}{q_2 - q_1}\right)$$

where $\Delta\theta \equiv \theta_{max} - \theta_{min}$.

Substituting the demand expressions in the brokers' profit equations:

$$\Pi_i = p_i D_i - \frac{q_i D_i}{Q_j} r_j - c_i D_i (Q_j - q_i)^2$$

and given Theorem 1 that B_1 buys service from S_1, and B_2 buys service from S_2, we obtain:

$$\Pi_1 = \frac{p_1}{\Delta\theta}\left(\frac{p_2 - p_1}{q_2 - q_1} - \theta_{min}\right) - \frac{q_1 r_1}{\Delta\theta Q_1}\left(\frac{p_2 - p_1}{q_2 - q_1} - \theta_{min}\right) - \frac{c(Q_1 - q_1)^2}{\Delta\theta}\left(\frac{p_2 - p_1}{q_2 - q_1} - \theta_{min}\right)$$

and

$$\Pi_2 = \frac{p_2}{\Delta\theta}\left(\theta_{\max} - \frac{p_2 - p_1}{q_2 - q_1}\right) - \frac{q_2 r_2}{\Delta\theta Q_2}\left(\theta_{\max} - \frac{p_2 - p_1}{q_2 - q_1}\right) - \frac{c(Q_2 - q_2)^2}{\Delta\theta}\left(\theta_{\max} - \frac{p_2 - p_1}{q_2 - q_1}\right).$$

To find the equilibrium in the broker-level game, we need to find the optimal price of brokers. To do so, we solve the $\partial\Pi_i / \partial p_i = 0$ system. Calculating

$\partial\Pi_i/\partial p_i$ for every i, we have:

$$\partial\Pi_1/\partial p_1 = \frac{1}{\Delta\theta}\left(\frac{p_2 - p_1}{q_2 - q_1} - \theta_{min}\right) - \frac{1}{(q_2 - q_1)} \times \frac{p_1}{\Delta\theta} + \frac{1}{(q_2 - q_1)} \times \frac{q_1 r_1}{\Delta\theta Q_1} + \frac{1}{(q_2 - q_1)} \times \frac{c(Q_1 - q_1)^2}{\Delta\theta}$$

and

$$\partial\Pi_2/\partial p_2 = \frac{1}{\Delta\theta}\left(\theta_{\max} - \frac{p_2 - p_1}{q_2 - q_1}\right) - \frac{1}{(q_2 - q_1)} \times \frac{p_2}{\Delta\theta} + \frac{1}{(q_2 - q_1)} \times \frac{q_2 r_2}{\Delta\theta Q_2} + \frac{1}{(q_2 - q_1)} \times \frac{c(Q_2 - q_2)^2}{\Delta\theta}.$$

Now, the solution of the system of two equations, *i.e.*, $\partial\Pi_i/\partial p_i = 0$, yields the optimal price for both brokers (shown in Equations (7) and (8)):

$$p_1 = \frac{1}{3}((q_2 - q_1)(\theta_{max} - 2\theta_{min}) + \frac{2q_1 r_1}{Q_1} + \frac{q_2 r_2}{Q_2} + 2c(q_1 - Q_1)^2 + c(q_2 - Q_2)^2)$$

$$p_2 = \frac{1}{3}((q_2 - q_1)(2\theta_{max} - \theta_{min}) + \frac{q_1 r_1}{Q_1} + \frac{2q_2 r_2}{Q_2} + c(q_1 - Q_1)^2 + 2c(q_2 - Q_2)^2)$$

Now the brokers' prices, p_1 and p_2, are a function of the brokers' and providers' service qualities, and providers' prices r_1 and r_2. The next step is to plug them into D_i's to obtain the demand as a function of r_j's (shown in Equations (9) and (10)):

$$D_1 = \frac{1}{3\Delta\theta}(\theta_{max} - 2\theta_{min}) + \frac{\frac{q_2 r_2}{Q_2} - \frac{q_1 r_1}{Q_1} - c(q_1 - Q_1)^2 + c(q_2 - Q_2)^2}{3\Delta\theta(q_2 - q_1)}$$

$$D_2 = \frac{1}{3\Delta\theta}(2\theta_{max} - \theta_{min}) + \frac{\frac{q_1 r_1}{Q_1} - \frac{q_2 r_2}{Q_2} + c(q_1 - Q_1)^2 - c(q_2 - Q_2)^2}{3\Delta\theta(q_2 - q_1)}$$

Now, D_1 and D_2 are a function of the service providers' prices r_1 and r_2. They can be substituted in the providers' profit and, following the same process, the optimal price of providers can be obtained in terms of providers' and brokers' service qualities.