

CS314: Algorithms

Homework 7

Required Problems

1. A boolean formula is in *disjunctive normal form* (or DNF) if it consists of a disjunction (OR) of several terms, each of which is the conjunction (AND) of one or more literals. For example, the formula:

$$(a \wedge b \wedge c) \vee (b \wedge c) \vee (a \wedge b \wedge c)$$

is in disjunctive normal form. DNF-SAT asks, given a boolean formula in disjunctive normal form, whether that formula is satisfiable.

- (a) Show that DNF-SAT is in P.
- (b) What is the error in the following argument that P=NP?

Suppose we are given a boolean formula in conjunctive normal form with at most three literals per clause, and we want to know if it is satisfiable. We can use the distributive law to construct an equivalent formula in disjunctive normal form. For example,

$$(a \vee b \vee c) \wedge (a \vee b) \iff (a \wedge b) \vee (b \wedge a) \vee (c \wedge a) \vee (c \wedge b)$$

Now we can use the algorithms from part (a) to determine, in polynomial time, whether the resulting DNF formula is satisfiable. We have just solved 3SAT in polynomial time! Since 3SAT is NP-hard, we must conclude that P=NP.

2. Note: for these problems, you may reference the descriptions of them from the lecture notes or (in the case of subset sum) from the textbook.
 - (a) Describe a polynomial time reduction from Partition to Subset Sum.
 - (b) Describe a polynomial time reduction from Subset Sum to Partition.