

Cache Memories

Authors:

Adapted from slides by Randy Bryant and Dave O'Hallaron

Chapter 6

- **SRAM vs. DRAM**
- Locality of reference
- Cache memory, organization, and operation
- Performance impact of caches
 - The memory mountain
 - Rearranging loops to improve spatial locality
 - Using blocking to improve temporal locality

Random-Access Memory (RAM)

■ Key features

- **RAM** is traditionally packaged as a chip.
- Basic storage unit is normally a **cell** (one bit per cell).
- Multiple RAM chips form a memory.

■ Static RAM (SRAM)

- Each cell stores a bit with a four or six-transistor circuit.
- Retains value indefinitely, as long as it is kept powered.
- Relatively insensitive to electrical noise (EMI), radiation, etc.
- Faster and more expensive than DRAM.

■ Dynamic RAM (DRAM)

- Each cell stores bit with a capacitor. One transistor is used for access
- Value must be refreshed every 10-100 ms.
- More sensitive to disturbances (EMI, radiation,...) than SRAM.
- Slower and cheaper than SRAM.

SRAM vs DRAM Summary

	Trans. per bit	Access time	Needs refresh?	Needs EDC?	Cost	Applications
SRAM	4 or 6	1X	No	Maybe	100x	Cache memories
DRAM	1	10X	Yes	Yes	1X	Main memories, frame buffers

Storage Trends

SRAM

Metric	1980	1985	1990	1995	2000	2005	2010	2010:1980
\$/MB	19,200	2,900	320	256	100	75	60	320
access (ns)	300	150	35	15	3	2	1.5	200

DRAM

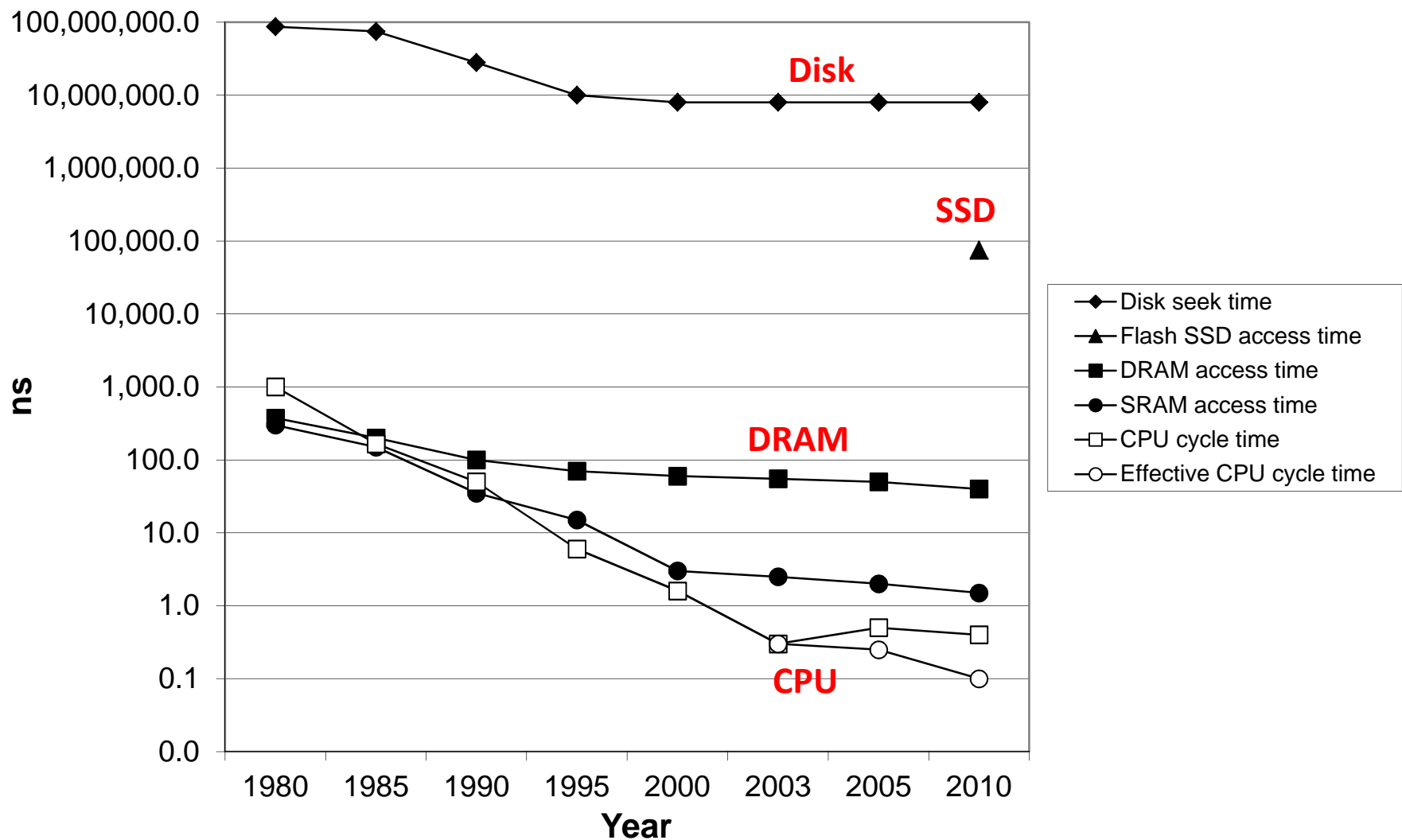
Metric	1980	1985	1990	1995	2000	2005	2010	2010:1980
\$/MB	8,000	880	100	30	1	0.1	0.06	130,000
access (ns)	375	200	100	70	60	50	40	9
typical size (MB)	0.064	0.256	4	16	64	2,000	8,000	125,000

Disk

Metric	1980	1985	1990	1995	2000	2005	2010	2010:1980
\$/MB	500	100	8	0.30	0.01	0.005	0.0003	1,600,000
access (ms)	87	75	28	10	8	4	3	29
typical size (MB)	1	10	160	1,000	20,000	160,000	1,500,000	1,500,000

The CPU-Memory Gap

The gap widens between DRAM, disk, and CPU speeds.



Locality to the Rescue!

The key to bridging this CPU-Memory gap is a fundamental property of computer programs known as **locality**

Chapter 6

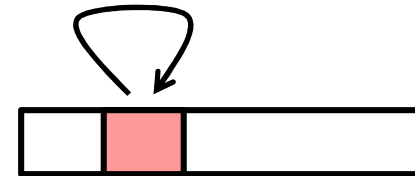
- SRAM vs. DRAM
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Locality

- **Principle of Locality:** Programs tend to use data and instructions with addresses near or equal to those they have used recently

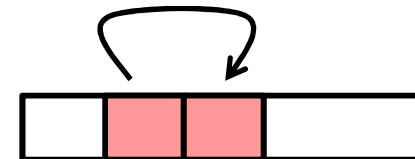
- **Temporal locality:**

- Recently referenced items are likely to be referenced again in the near future



- **Spatial locality:**

- Items with nearby addresses tend to be referenced close together in time



Locality Example

```
sum = 0;  
for (i = 0; i < n; i++)  
    sum += a[i];  
return sum;
```

■ Data references

- Reference array elements in succession (stride-1 reference pattern).
- Reference variable `sum` each iteration.

Spatial locality

Temporal locality

■ Instruction references

- Reference instructions in sequence.
- Cycle through loop repeatedly.

Spatial locality

Temporal locality

Qualitative Estimates of Locality

- **Claim:** Being able to look at code and get a qualitative sense of its locality is a key skill for a professional programmer.
- **Question:** Does this function have good locality with respect to array *a*?

```
int sum_array_rows(int a[M][N])
{
    int i, j, sum = 0;

    for (i = 0; i < M; i++)
        for (j = 0; j < N; j++)
            sum += a[i][j];
    return sum;
}
```

Locality Example

- **Question:** Does this function have good locality with respect to array *a*?

```
int sum_array_cols(int a[M][N])
{
    int i, j, sum = 0;

    for (j = 0; j < N; j++)
        for (i = 0; i < M; i++)
            sum += a[i][j];
    return sum;
}
```

Locality Example

- **Question:** Can you permute the loops so that the function scans the 3-d array `a` with a stride-1 reference pattern (and thus has good spatial locality)?

```
int sum_array_3d(int a[M][N][N])
{
    int i, j, k, sum = 0;

    for (i = 0; i < M; i++)
        for (j = 0; j < N; j++)
            for (k = 0; k < N; k++)
                sum += a[k][i][j];

    return sum;
}
```

Memory Hierarchies

- **Some fundamental and enduring properties of hardware and software:**
 - Fast storage technologies cost more per byte, have less capacity, and require more power (heat!).
 - The gap between CPU and main memory speed is widening.
 - Well-written programs tend to exhibit good locality.
- **These fundamental properties complement each other beautifully.**
- **They suggest an approach for organizing memory and storage systems known as a **memory hierarchy**.**

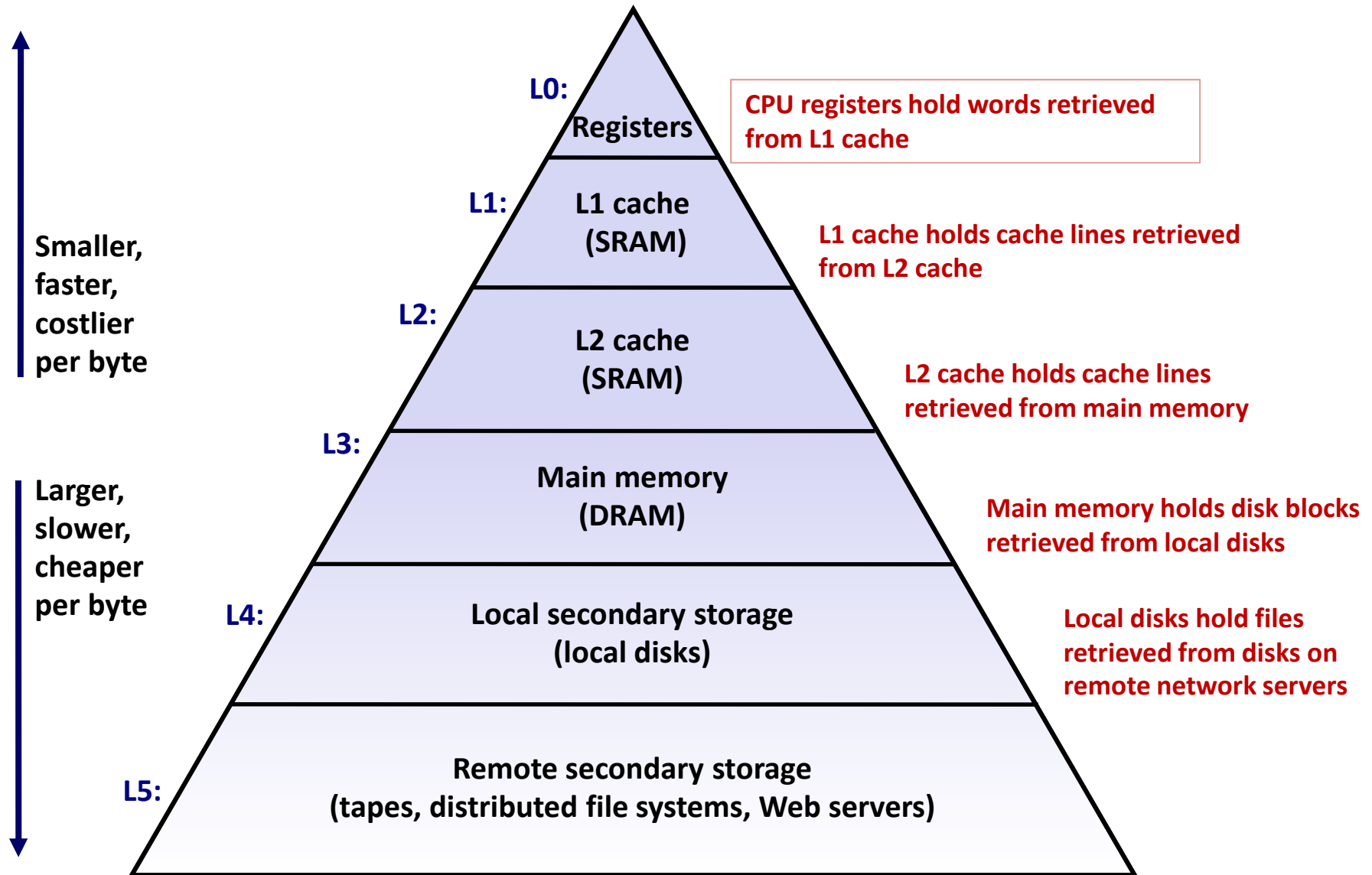
Chapter 6

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Caches

- **Cache:** A smaller, faster storage device that acts as a staging area for a subset of the data in a larger, slower device.
- **Fundamental idea of a memory hierarchy:**
 - For each k , the faster, smaller device at level k serves as a cache for the larger, slower device at level $k+1$.
- **Why do memory hierarchies work?**
 - Because of locality, programs tend to access the data at level k more often than they access the data at level $k+1$.
 - Thus, the storage at level $k+1$ can be slower, and thus larger and cheaper per bit.
- **Big Idea:** The memory hierarchy creates a large pool of storage that costs as much as the cheap storage near the bottom, but that serves data to programs at the rate of the fast storage near the top.

An Example Memory Hierarchy

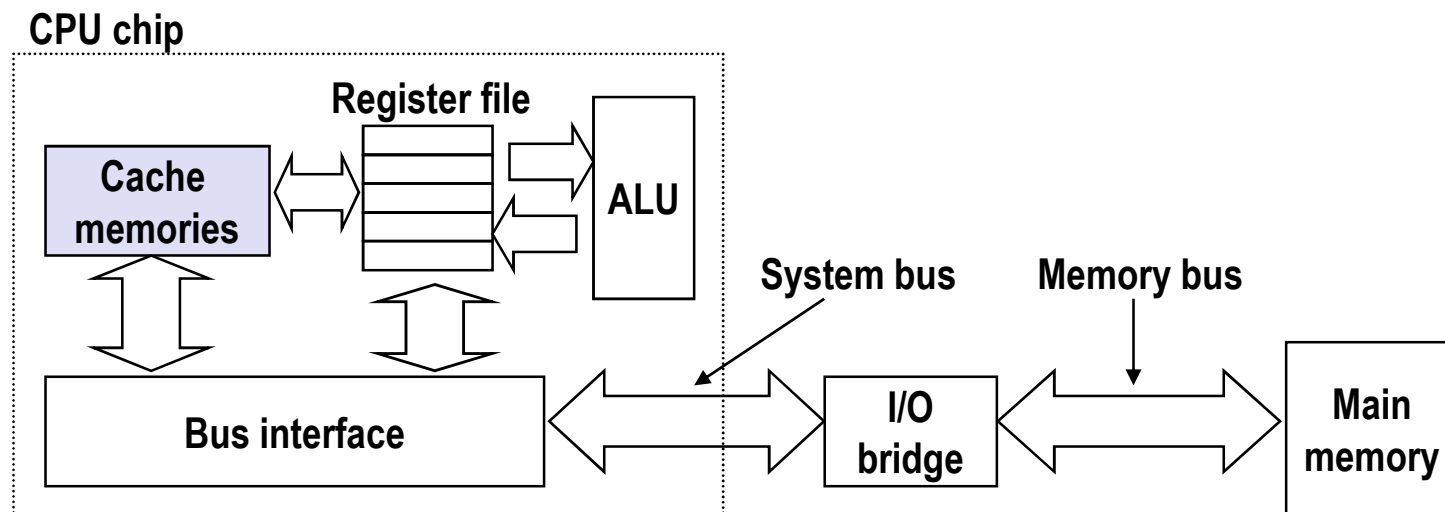


Examples of Caching in the Hierarchy

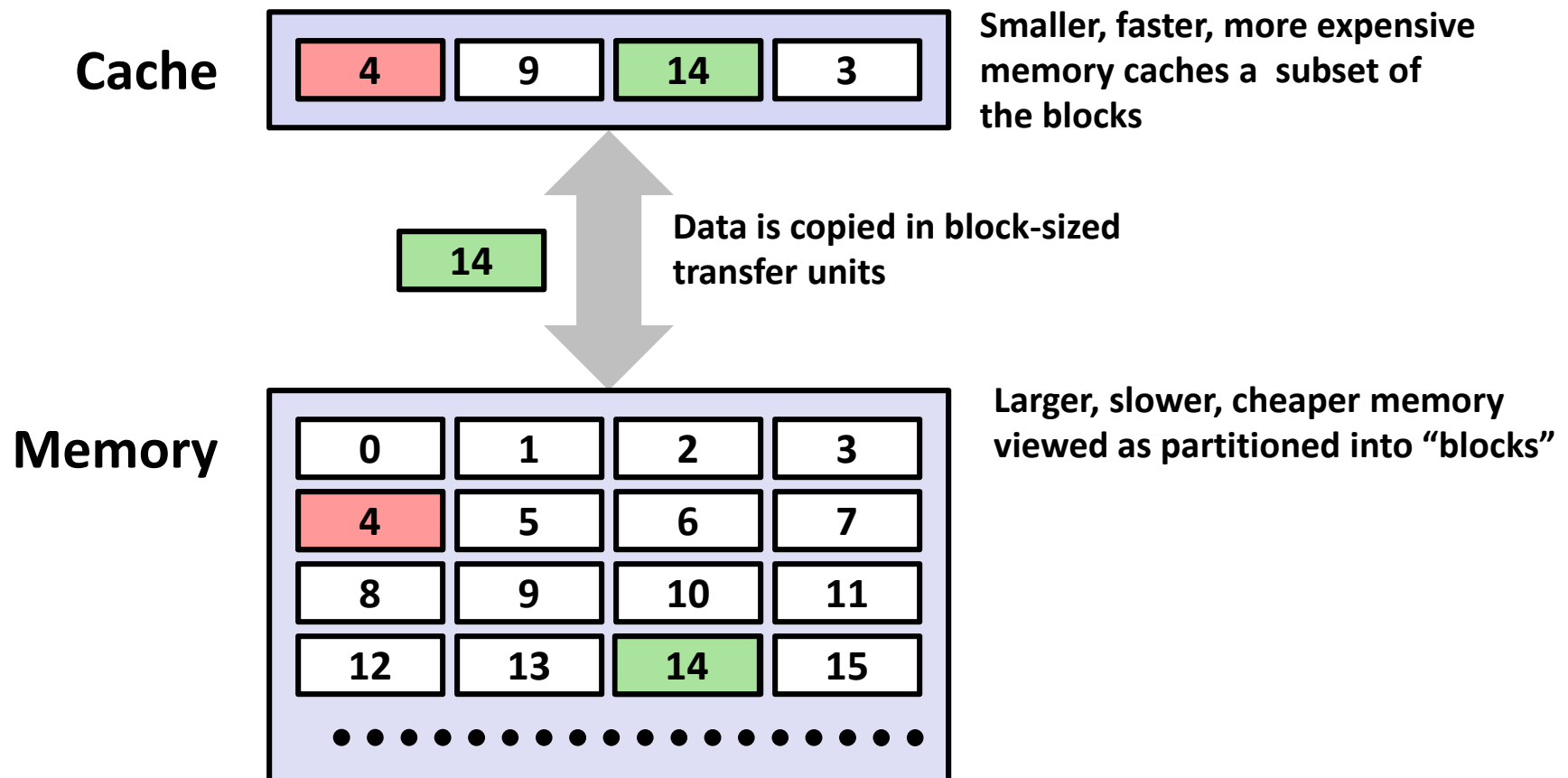
Cache Type	What is Cached?	Where is it Cached?	Latency (cycles)	Managed By
Registers	4-8 bytes words	CPU core	0	Compiler
TLB	Address translations	On-Chip TLB	0	Hardware
L1 cache	64-bytes block	On-Chip L1	1	Hardware
L2 cache	64-bytes block	On/Off-Chip L2	10	Hardware
Virtual Memory	4-KB page	Main memory	100	Hardware + OS
Buffer cache	Parts of files	Main memory	100	OS
Disk cache	Disk sectors	Disk controller	100,000	Disk firmware
Network buffer cache	Parts of files	Local disk	10,000,000	AFS/NFS client
Browser cache	Web pages	Local disk	10,000,000	Web browser
Web cache	Web pages	Remote server disks	1,000,000,000	Web proxy server

Cache Memories

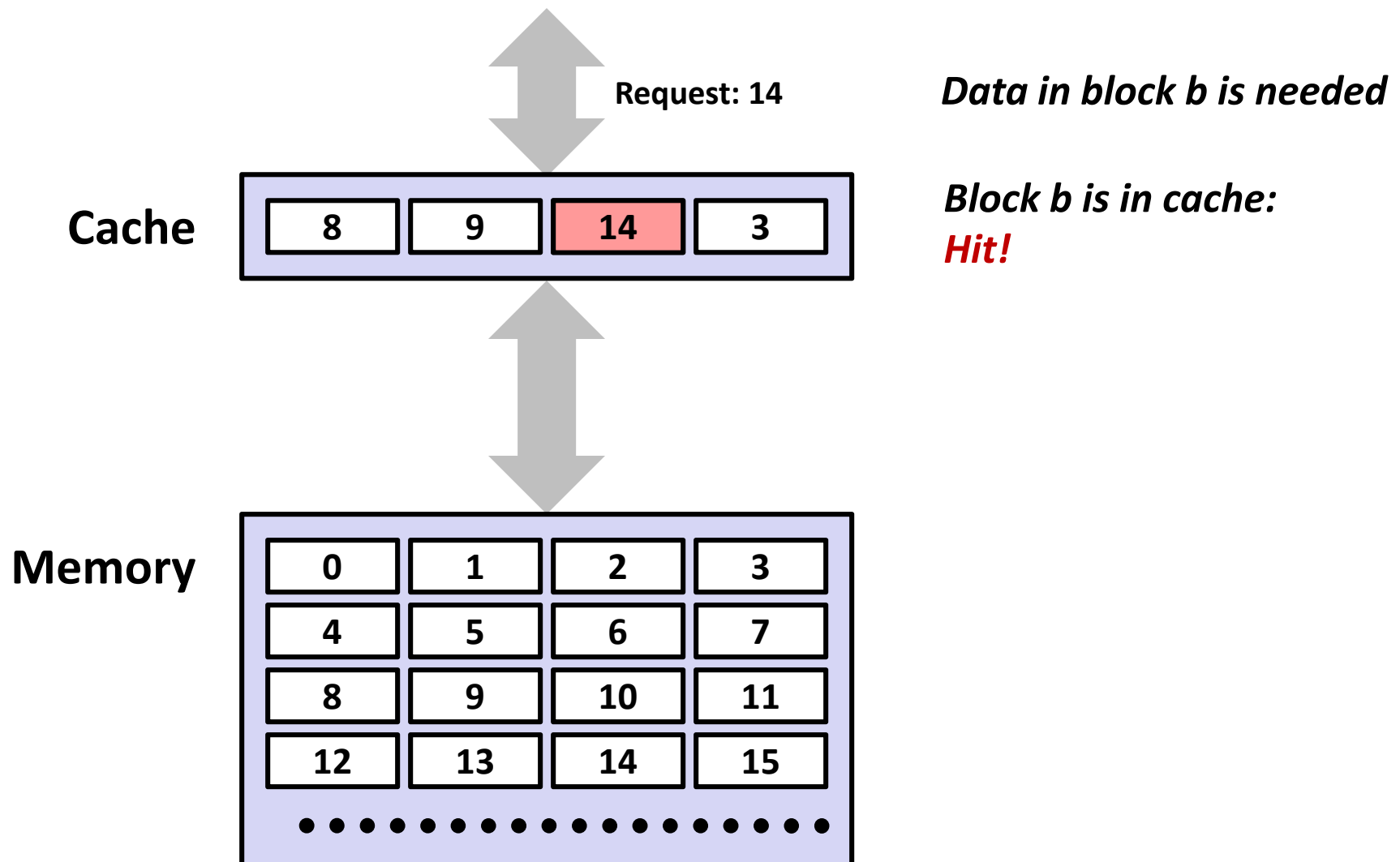
- **Cache memories** are small, fast SRAM-based memories managed automatically in hardware.
 - Hold frequently accessed blocks of main memory
- CPU looks first for data in caches (e.g., L1, L2, and L3), then in main memory.
- Typical system structure:



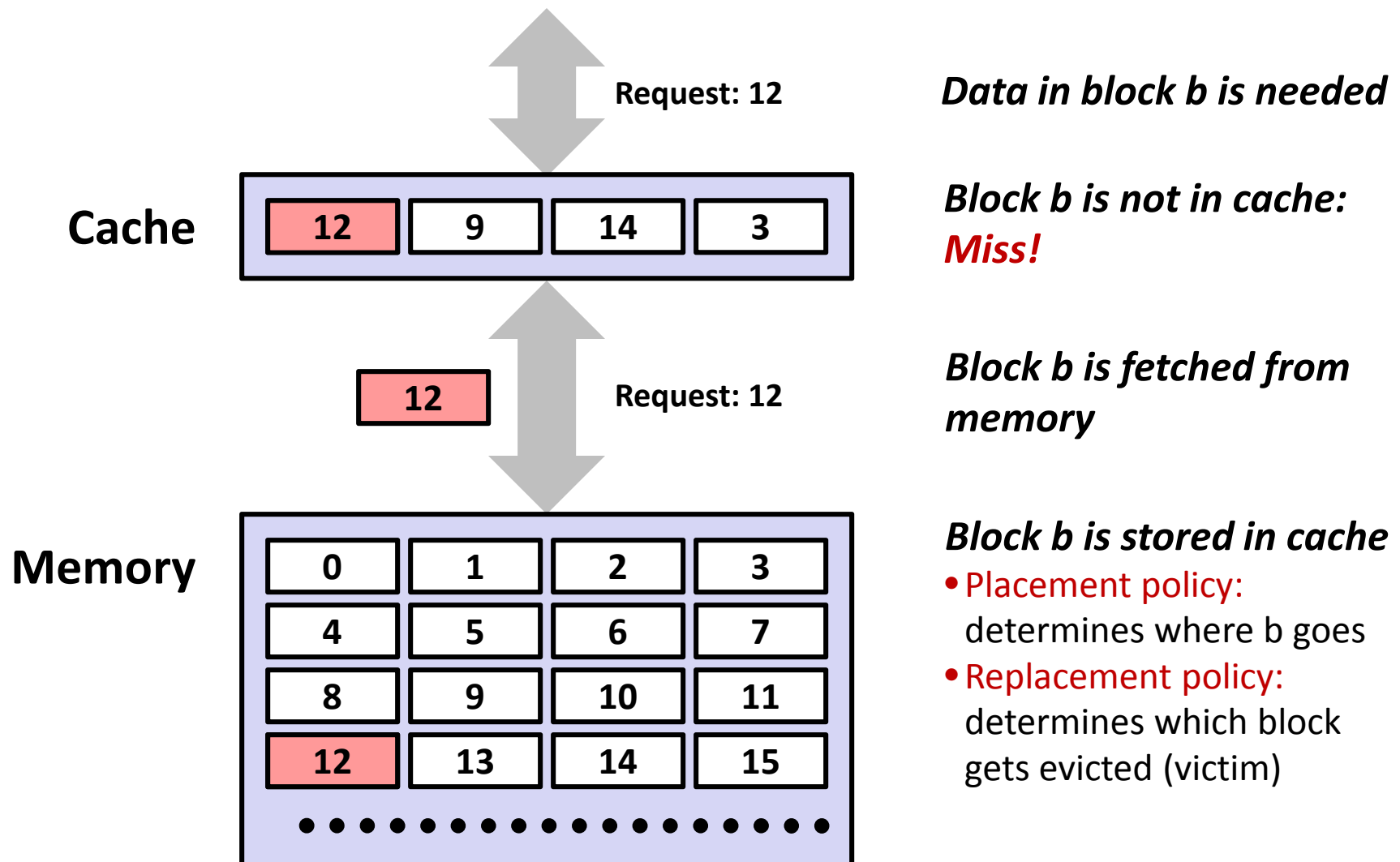
General Cache Concepts



General Cache Concepts: Hit



General Cache Concepts: Miss



General Caching Concepts:

Types of Cache Misses

■ Cold (compulsory) miss

- Cold misses occur because the cache is empty.

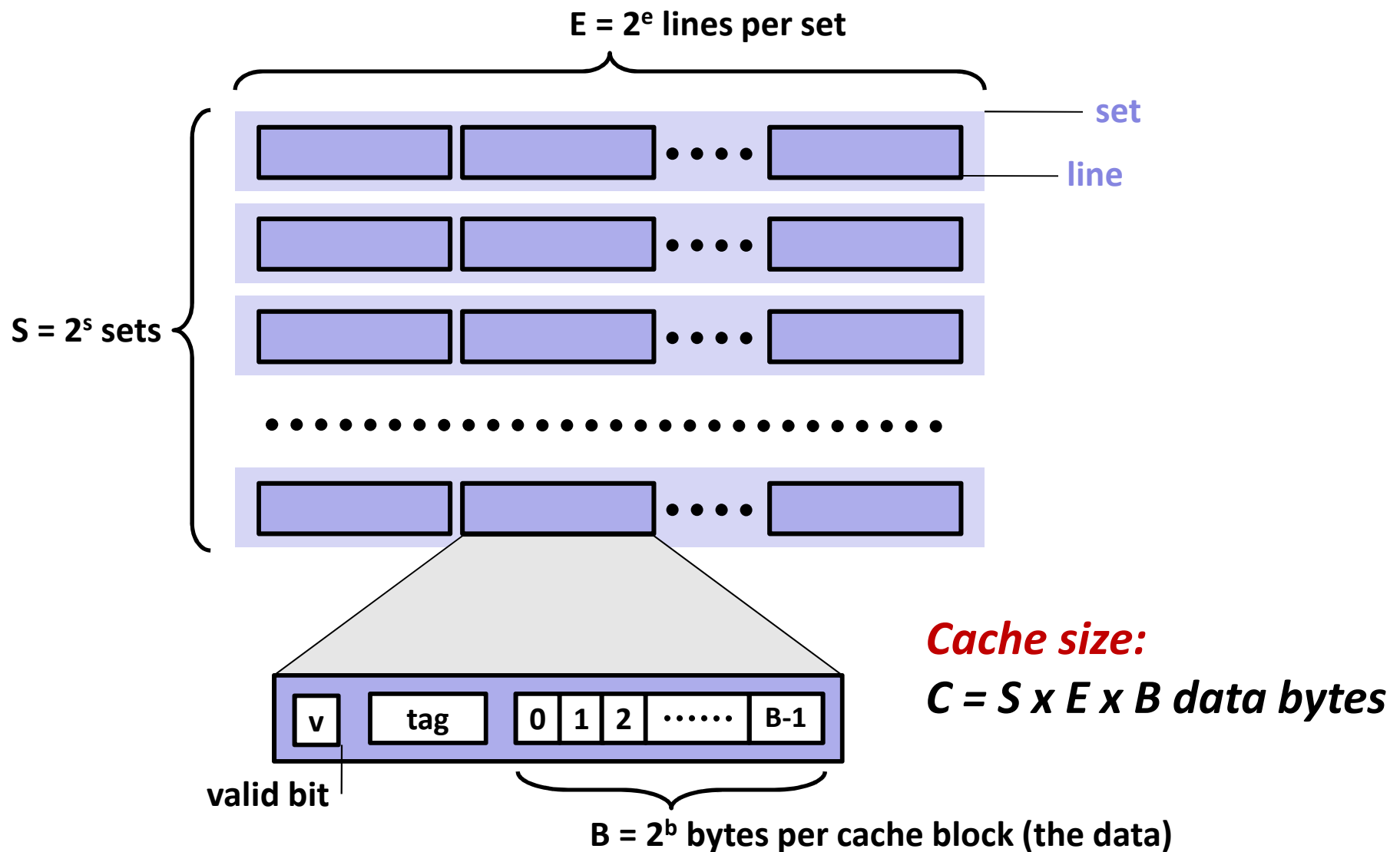
■ Conflict miss

- Most caches limit blocks at level $k+1$ to a small subset (sometimes a singleton) of the block positions at level k .
 - E.g. Block i at level $k+1$ must be placed in block $(i \bmod 4)$ at level k .
- Conflict misses occur when the level k cache is large enough, but multiple data objects all map to the same level k block.
 - E.g. Referencing blocks 0, 8, 0, 8, 0, 8, ... would miss every time.

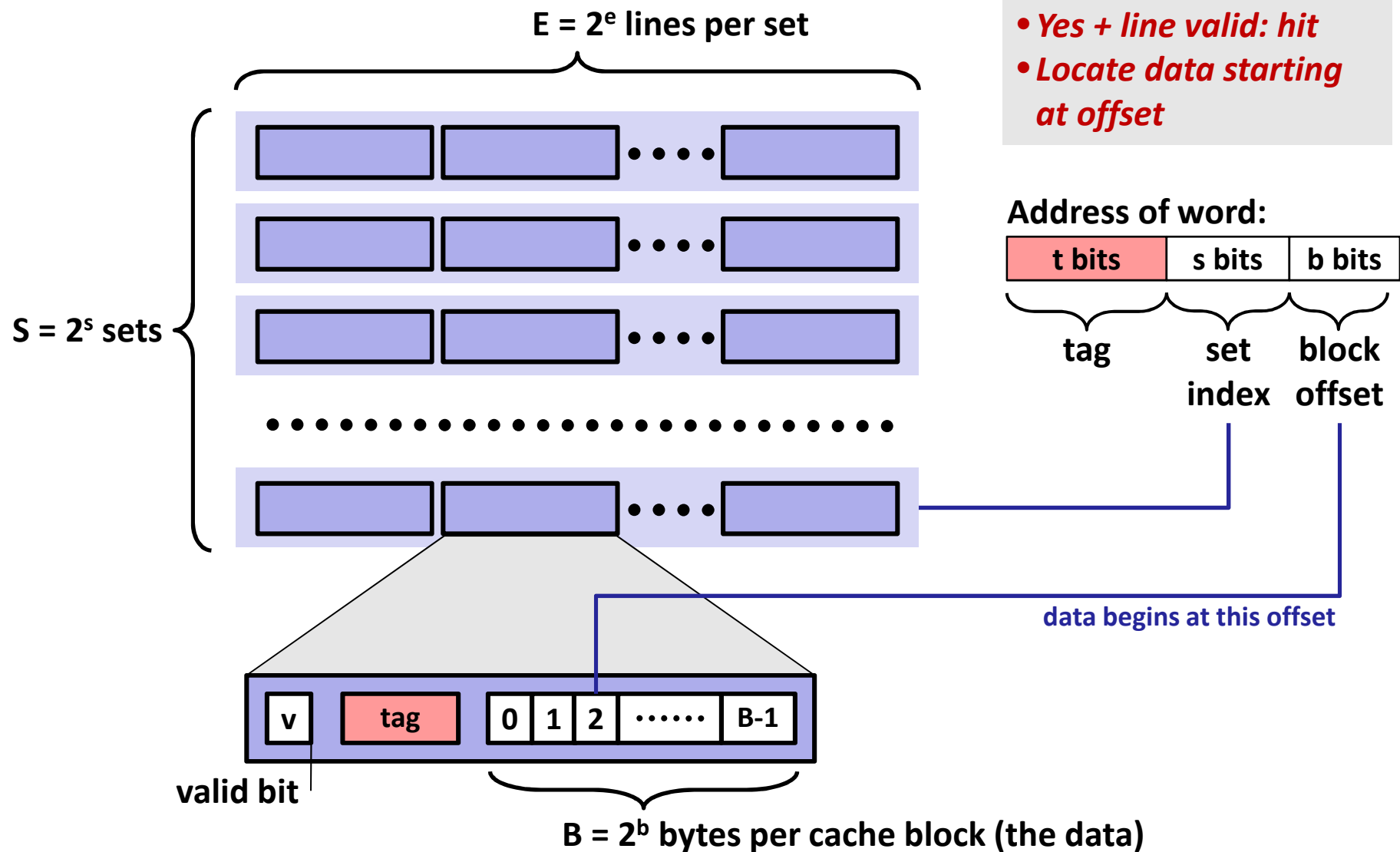
■ Capacity miss

- Occurs when the set of active cache blocks (**working set**) is larger than the cache.

General Cache Organization (S, E, B)



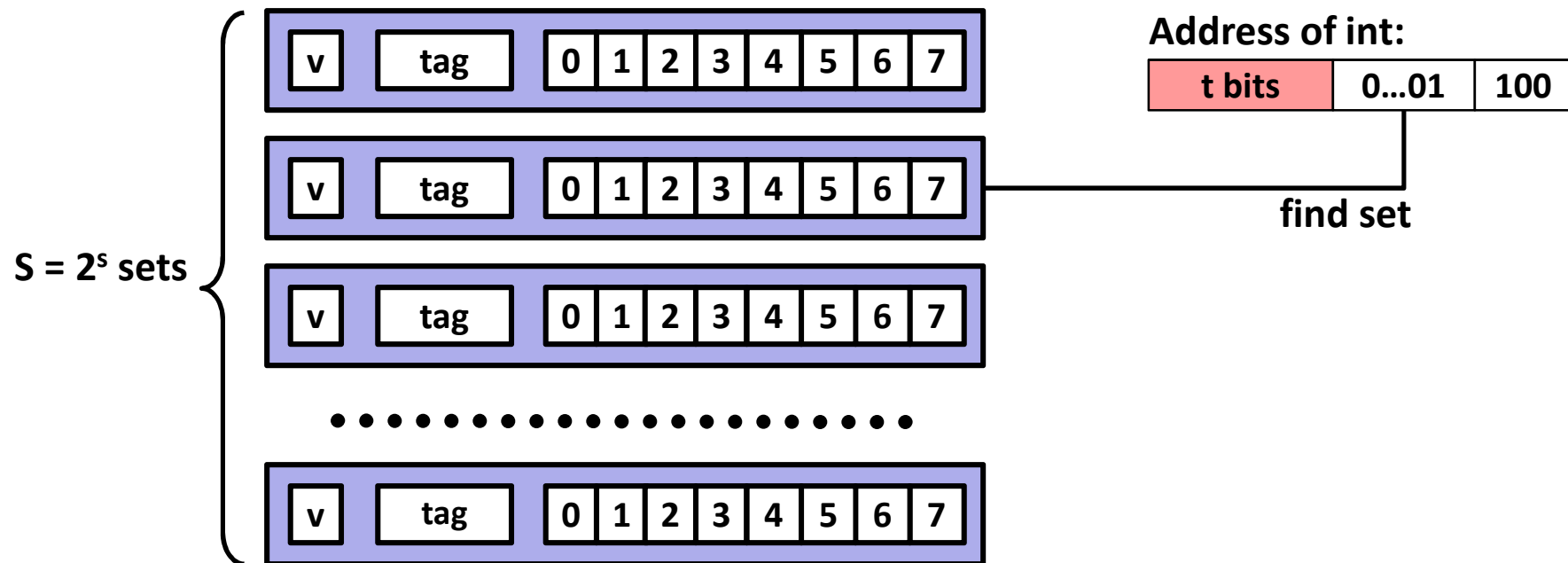
Cache Read



Example: Direct Mapped Cache (E = 1)

Direct mapped: One line per set

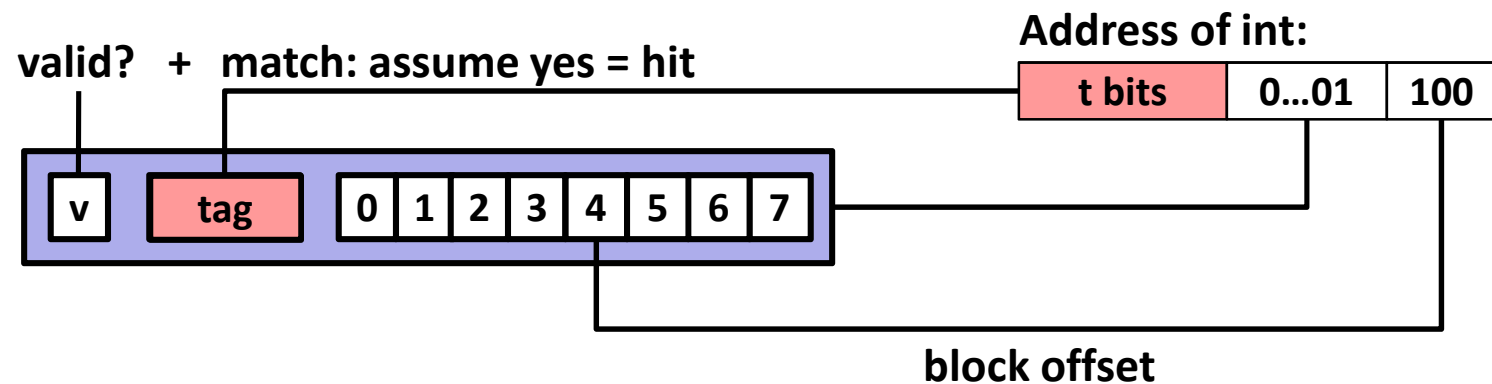
Assume: cache block size 8 bytes



Example: Direct Mapped Cache (E = 1)

Direct mapped: One line per set

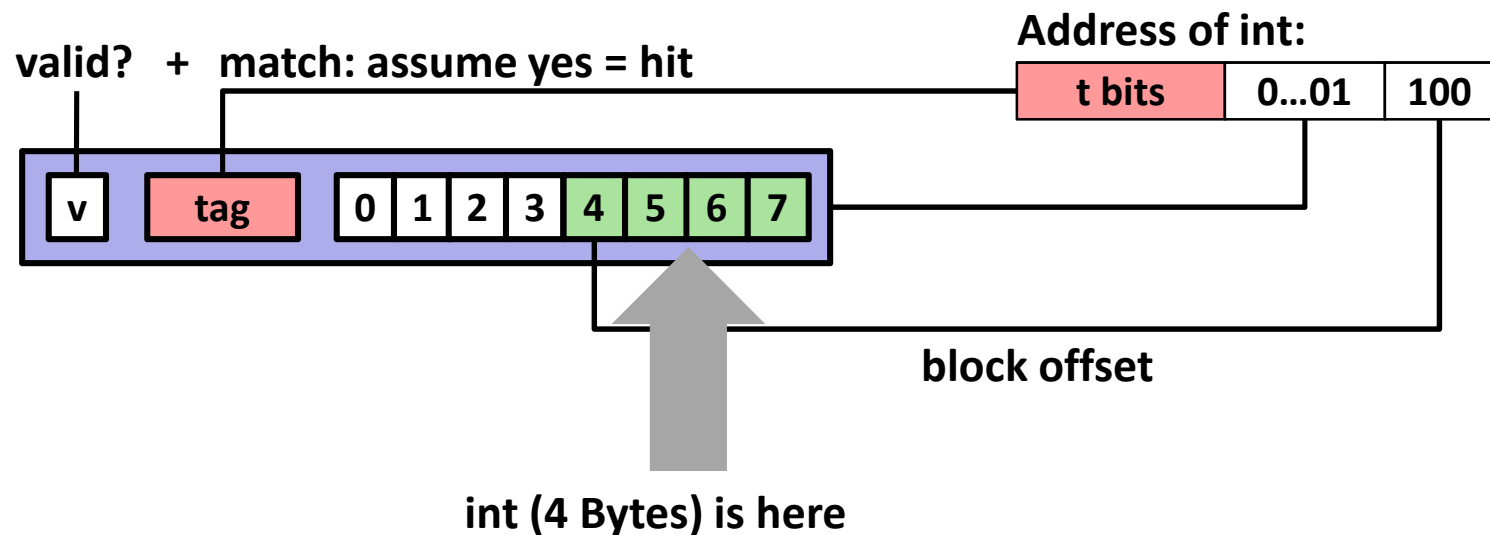
Assume: cache block size 8 bytes



Example: Direct Mapped Cache (E = 1)

Direct mapped: One line per set

Assume: cache block size 8 bytes



No match: old line is evicted and replaced

Direct-Mapped Cache Simulation

M = 16 byte addresses

B = 2 bytes/block,

S = 4 sets

E = 1 block/set

t=1	s=2	b=1
x	xx	x

Address trace (reads, one byte per read):

0	[<u>000</u> ₂]	miss
1	[<u>001</u> ₂]	hit
7	[<u>011</u> ₂]	miss
8	[<u>100</u> ₂]	miss
0	[<u>000</u> ₂]	miss
A	[<u>101</u> ₂]	miss
6	[<u>011</u> ₂]	hit

Initial cache configuration:

	v	Tag	Block
Set 0	1	1	M[8-9]
Set 1	0		
Set 2	0		
Set 3	0		

Final cache configuration:

	v	Tag	Block
Set 0	1	0	M[0-1]
Set 1	1	1	M[10-11]
Set 2	0		
Set 3	1	0	M[6-7]

A Higher Level Example

```
int sum_array_rows(double a[16][16])
{
    int i, j;
    double sum = 0;

    for (i = 0; i < 16; i++)
        for (j = 0; j < 16; j++)
            sum += a[i][j];
    return sum;
}
```

```
int sum_array_cols(double a[16][16])
{
    int i, j;
    double sum = 0;

    for (j = 0; j < 16; j++)
        for (i = 0; i < 16; i++)
            sum += a[i][j];
    return sum;
}
```

Assume:
one block per set, and
8 doubles per block

assume: cold (empty) cache,
a[0][0] goes here



64 B ∴ 8 doubles

Ignore the variables sum, i, j

E-way Set Associative Cache (Here: E = 2)

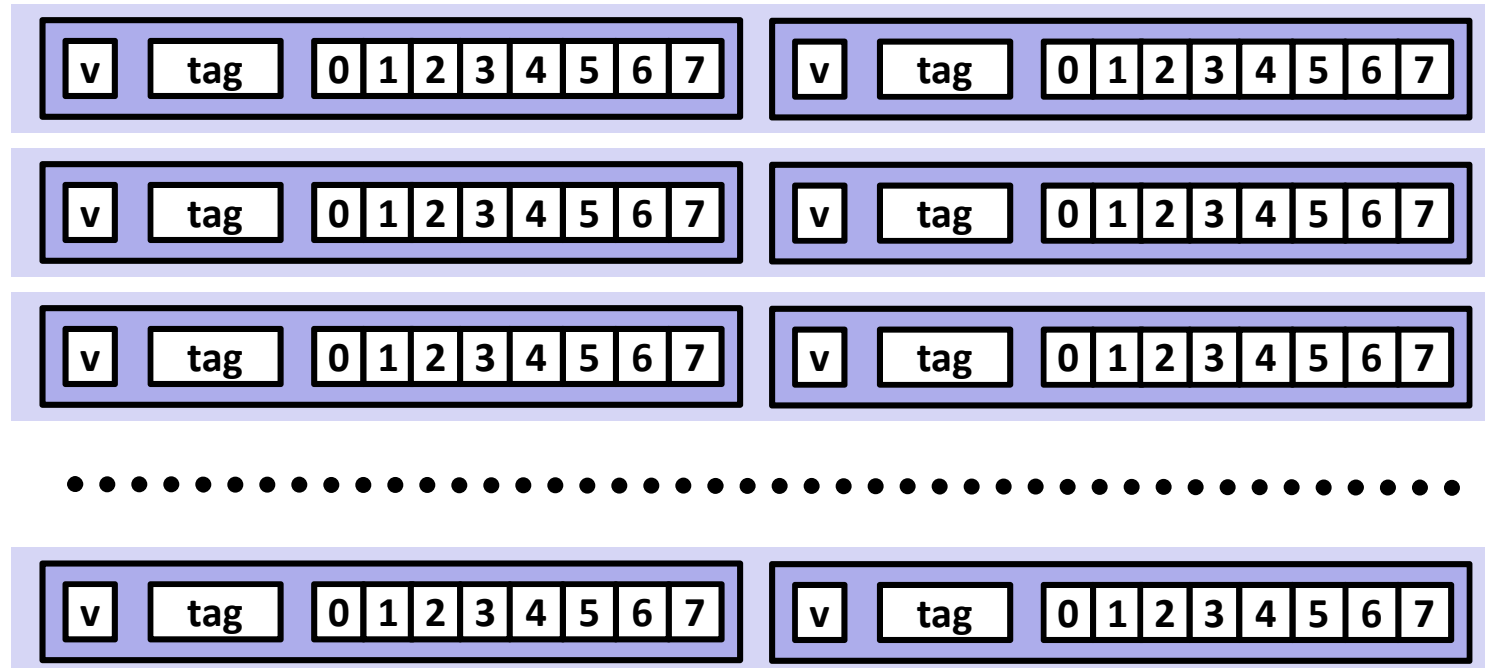
E = 2: Two lines per set

Assume: cache block size 8 bytes

Address of short int:

t bits	0...01	100
--------	--------	-----

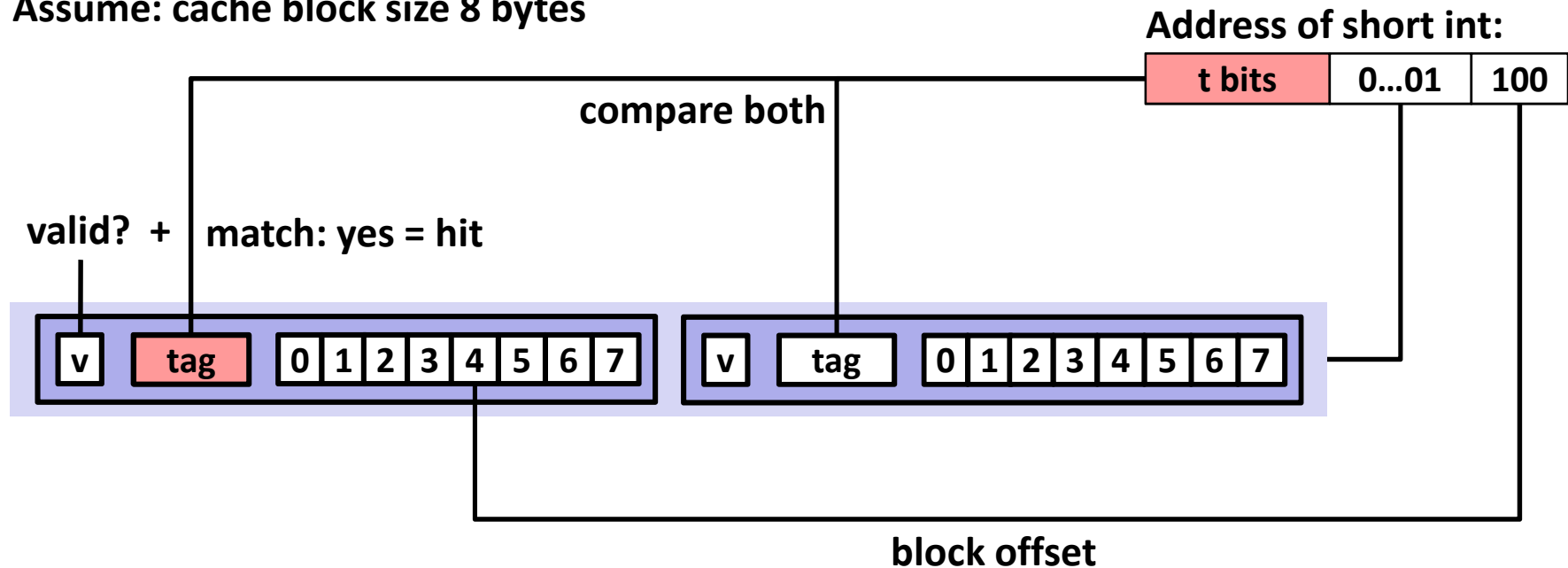
find set



E-way Set Associative Cache (Here: E = 2)

E = 2: Two lines per set

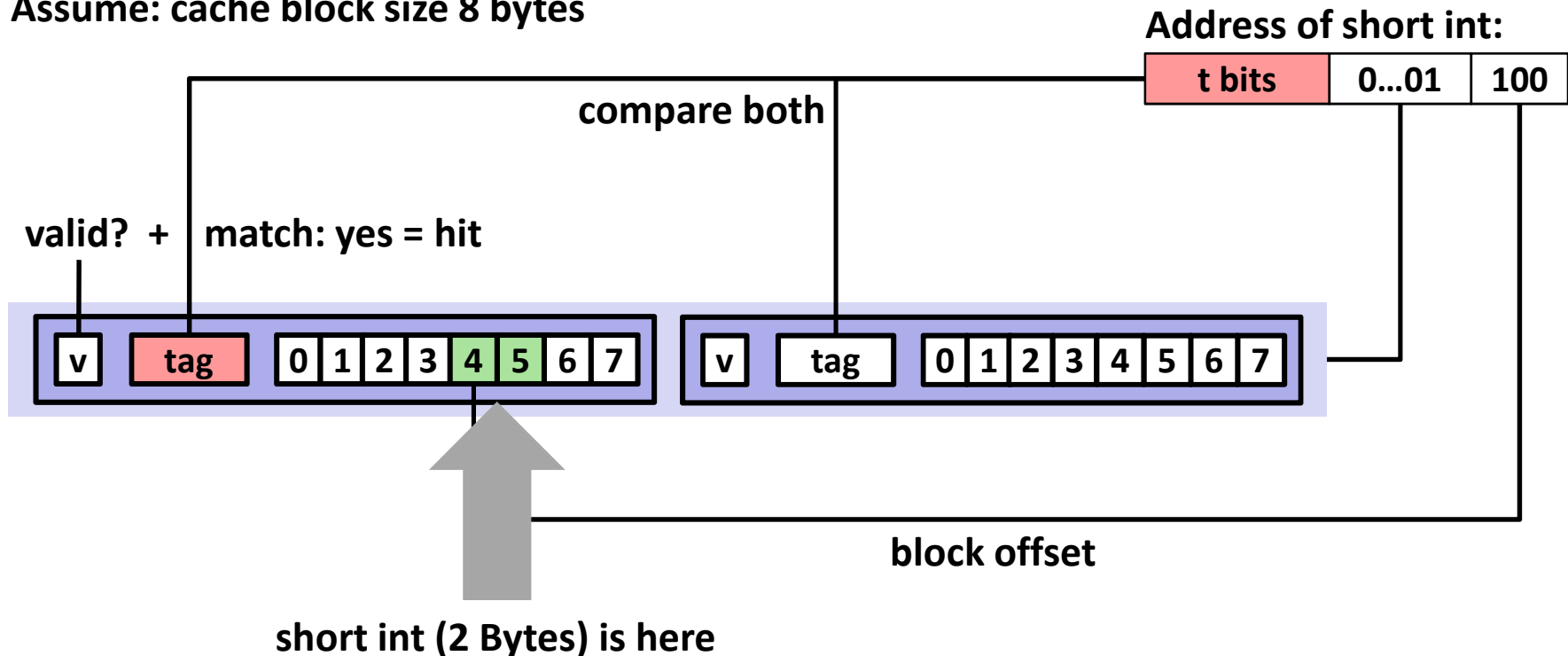
Assume: cache block size 8 bytes



E-way Set Associative Cache (Here: E = 2)

E = 2: Two lines per set

Assume: cache block size 8 bytes



No match:

- One line in set is selected for eviction and replacement
- Replacement policies: random, least recently used (LRU), ...

2-Way Set Associative Cache Simulation

M = 16 byte addresses

B = 2 bytes/block,

S = 4 sets

E = 1 block/set

t=2	s=1	b=1
xx	x	x

Address trace (reads, one byte per read):

0	[00 <u>0</u> 0 ₂]	miss
1	[00 <u>0</u> 1 ₂]	hit
7	[01 <u>1</u> 1 ₂]	miss
8	[10 <u>0</u> 0 ₂]	hit
0	[00 <u>0</u> 0 ₂]	hit
A	[10 <u>1</u> 0 ₂]	miss
6	[01 <u>1</u> 0 ₂]	hit

Initial cache configuration:

	v	Tag	Block
Set 0	0		
	1	10	M[8-9]
Set 1	0		
	0		

Final cache configuration:

	v	Tag	Block
Set 0	1	00	M[0-1]
	1	10	M[8-9]
Set 1	1	01	M[6-7]
	1	10	M[10-11]

A Higher Level Example

```
int sum_array_rows(double a[16][16])
{
    int i, j;
    double sum = 0;

    for (i = 0; i < 16; i++)
        for (j = 0; j < 16; j++)
            sum += a[i][j];
    return sum;
}
```

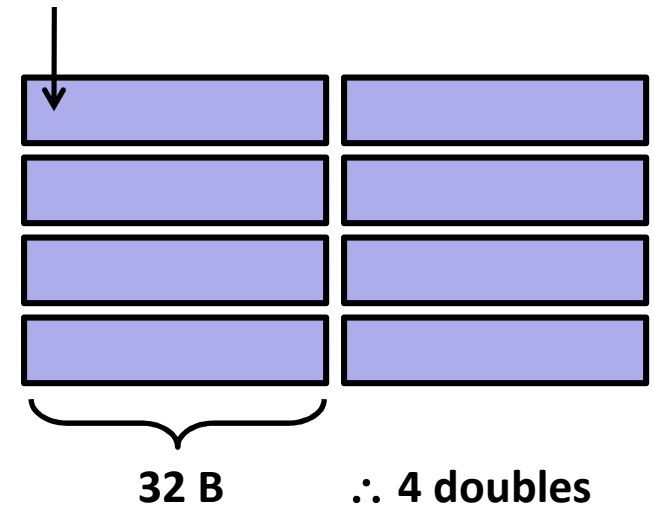
```
int sum_array_rows(double a[16][16])
{
    int i, j;
    double sum = 0;

    for (j = 0; j < 16; j++)
        for (i = 0; i < 16; i++)
            sum += a[i][j];
    return sum;
}
```

Assume:

two blocks per set, but
only **4 doubles** per block

assume: cold (empty) cache,
a[0][0] goes here



Ignore the variables sum, i, j

What about writes?

■ Multiple copies of data exist:

- L1, L2, Main Memory, Disk

■ What to do on a write-hit?

- **Write-through** (write immediately to memory)
- **Write-back** (defer write to memory until replacement of line)
 - Need a dirty bit (line different from memory or not)

■ What to do on a write-miss?

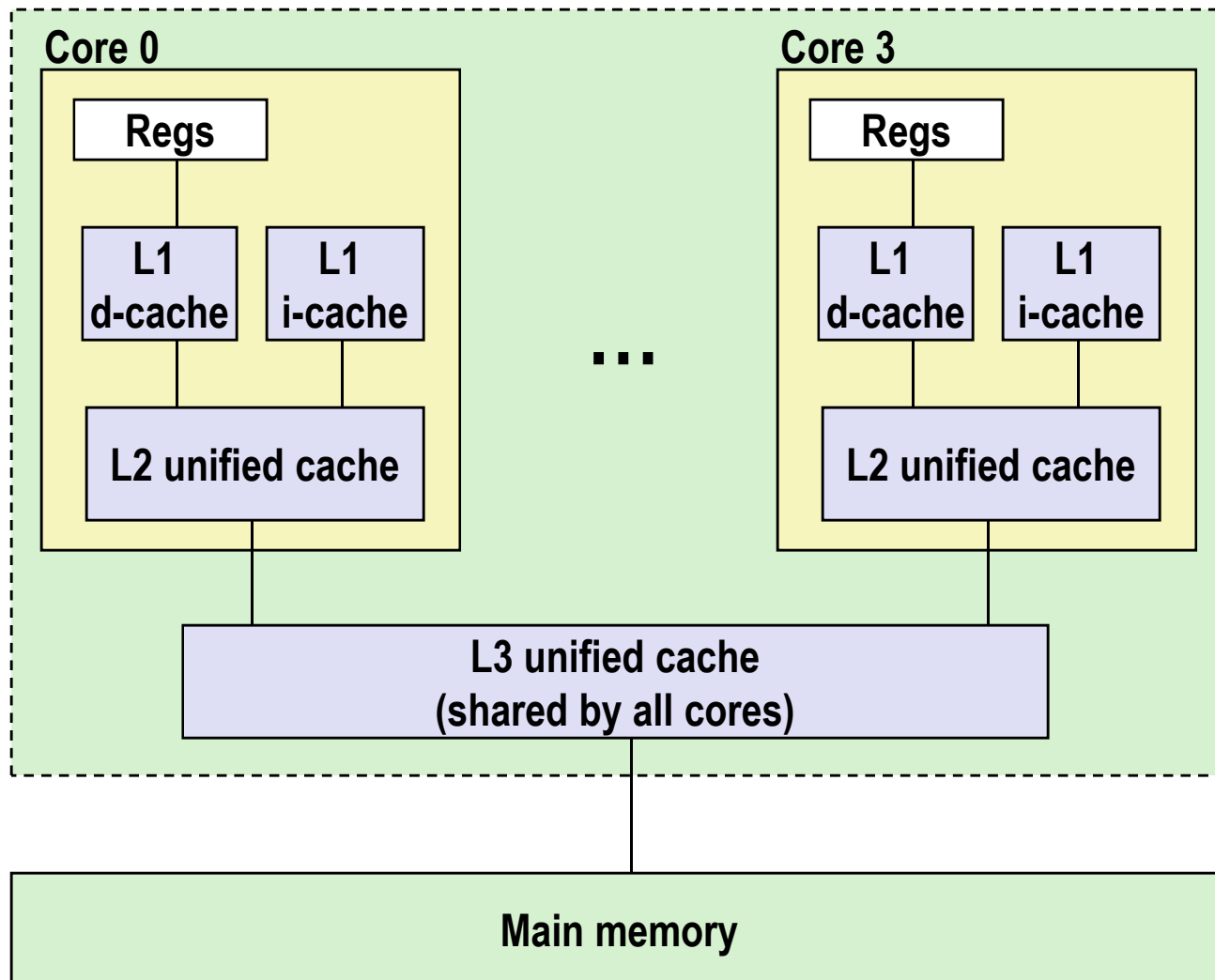
- **Write-allocate** (load into cache, update line in cache)
 - Good if more writes to the location follow
- **No-write-allocate** (writes immediately to memory)

■ Typical

- Write-through + No-write-allocate
- **Write-back + Write-allocate**

Intel Core i7 Cache Hierarchy

Processor package



L1 i-cache and d-cache:
32 KB, 8-way,
Access: 4 cycles

L2 unified cache:
256 KB, 8-way,
Access: 11 cycles

L3 unified cache:
8 MB, 16-way,
Access: 30-40 cycles

Block size: 64 bytes for
all caches.

Cache Performance Metrics

■ Miss Rate

- Fraction of memory references not found in cache (misses / accesses)
= $1 - \text{hit rate}$
- Typical numbers (in percentages):
 - 3-10% for L1
 - can be quite small (e.g., $< 1\%$) for L2, depending on size, etc.

■ Hit Time

- Time to deliver a line in the cache to the processor
 - includes time to determine whether the line is in the cache
- Typical numbers:
 - 1-2 clock cycle for L1
 - 5-20 clock cycles for L2

■ Miss Penalty

- Additional time required because of a miss
 - typically 50-200 cycles for main memory (Trend: increasing!)

Lets think about those numbers

- **Huge difference between a hit and a miss**
 - Could be 100x, if just L1 and main memory

- **Would you believe 99% hits is twice as good as 97%?**
 - Consider:
cache hit time of 1 cycle
miss penalty of 100 cycles

 - Average access time:
97% hits: $1 \text{ cycle} + 0.03 * 100 \text{ cycles} = 4 \text{ cycles}$
99% hits: $1 \text{ cycle} + 0.01 * 100 \text{ cycles} = 2 \text{ cycles}$

- **This is why “miss rate” is used instead of “hit rate”**

Writing Cache Friendly Code

- **Make the common case go fast**
 - Focus on the inner loops of the core functions
- **Minimize the misses in the inner loops**
 - Repeated references to variables are good (**temporal locality**)
 - Stride-1 reference patterns are good (**spatial locality**)

Key idea: Our qualitative notion of locality is quantified through our understanding of cache memories.

Today

- SRAM vs. DRAM
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The Memory Mountain

- **Read throughput** (read bandwidth)
 - Number of bytes read from memory per second (MB/s)
- **Memory mountain:** Measured read throughput as a function of spatial and temporal locality.
 - Compact way to characterize memory system performance.

Memory Mountain Test Function

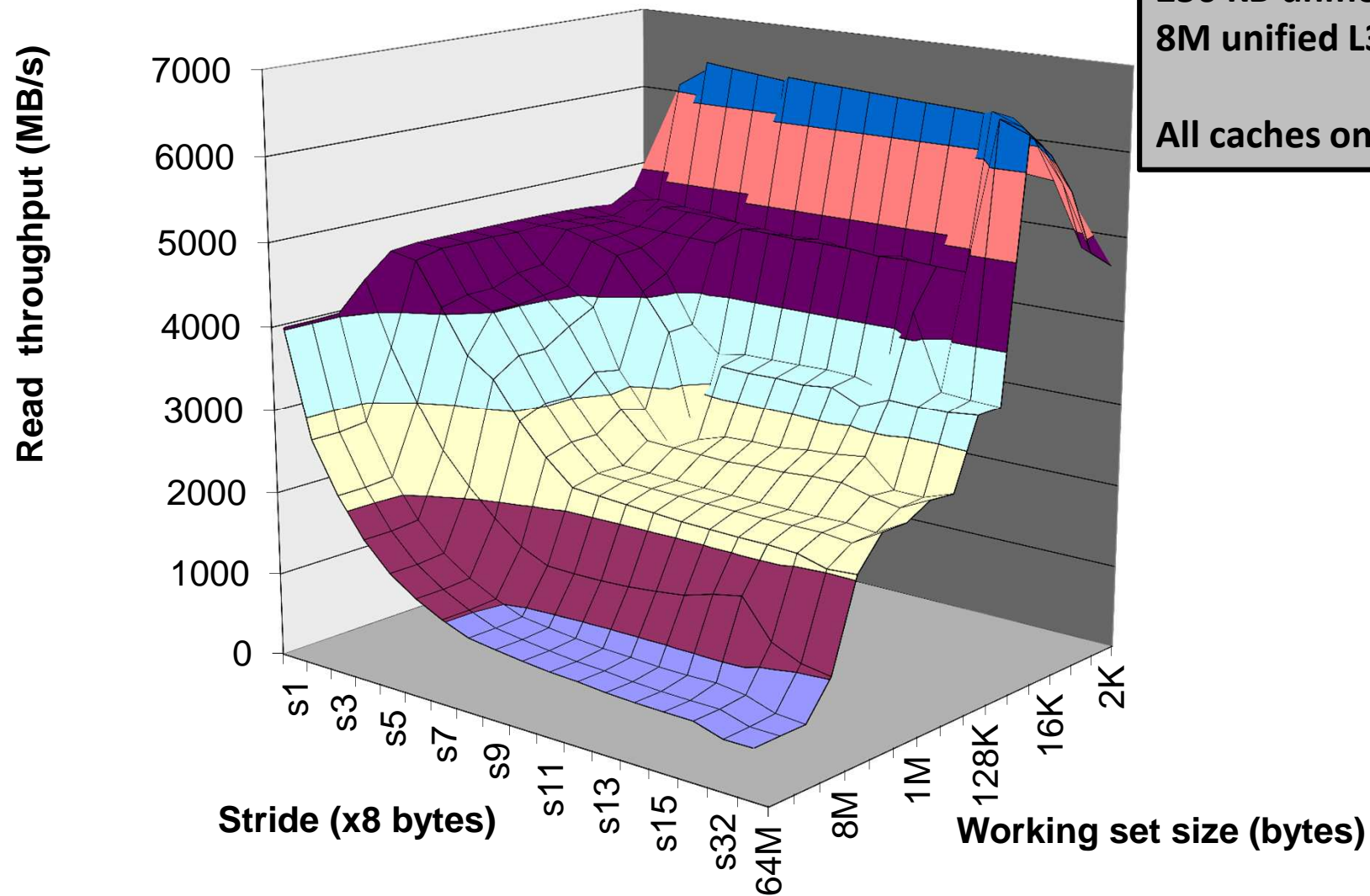
```
/* The test function */
void test(int elems, int stride) {
    int i, result = 0;
    volatile int sink;

    for (i = 0; i < elems; i += stride)
        result += data[i];
    sink = result; /* So compiler doesn't optimize away the loop */
}

/* Run test(elems, stride) and return read throughput (MB/s) */
double run(int size, int stride, double Mhz)
{
    double cycles;
    int elems = size / sizeof(int);

    test(elems, stride); /* warm up the cache */
    cycles = fcyc2(test, elems, stride, 0); /* call test(elems, stride) */
    return (size / stride) / (cycles / Mhz); /* convert cycles to MB/s */
}
```

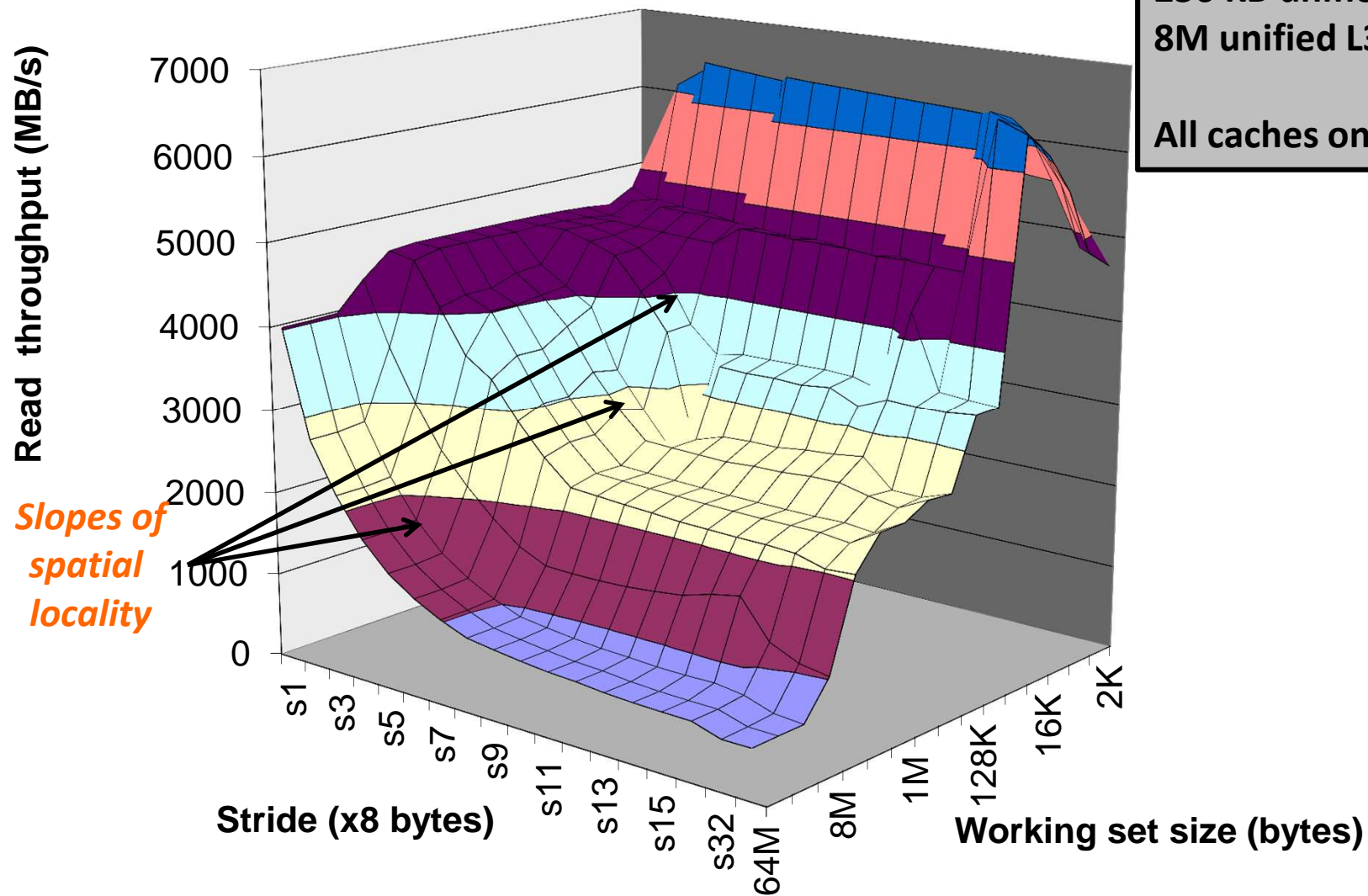
The Memory Mountain



Intel Core i7
 32 KB L1 i-cache
 32 KB L1 d-cache
 256 KB unified L2 cache
 8M unified L3 cache

All caches on-chip

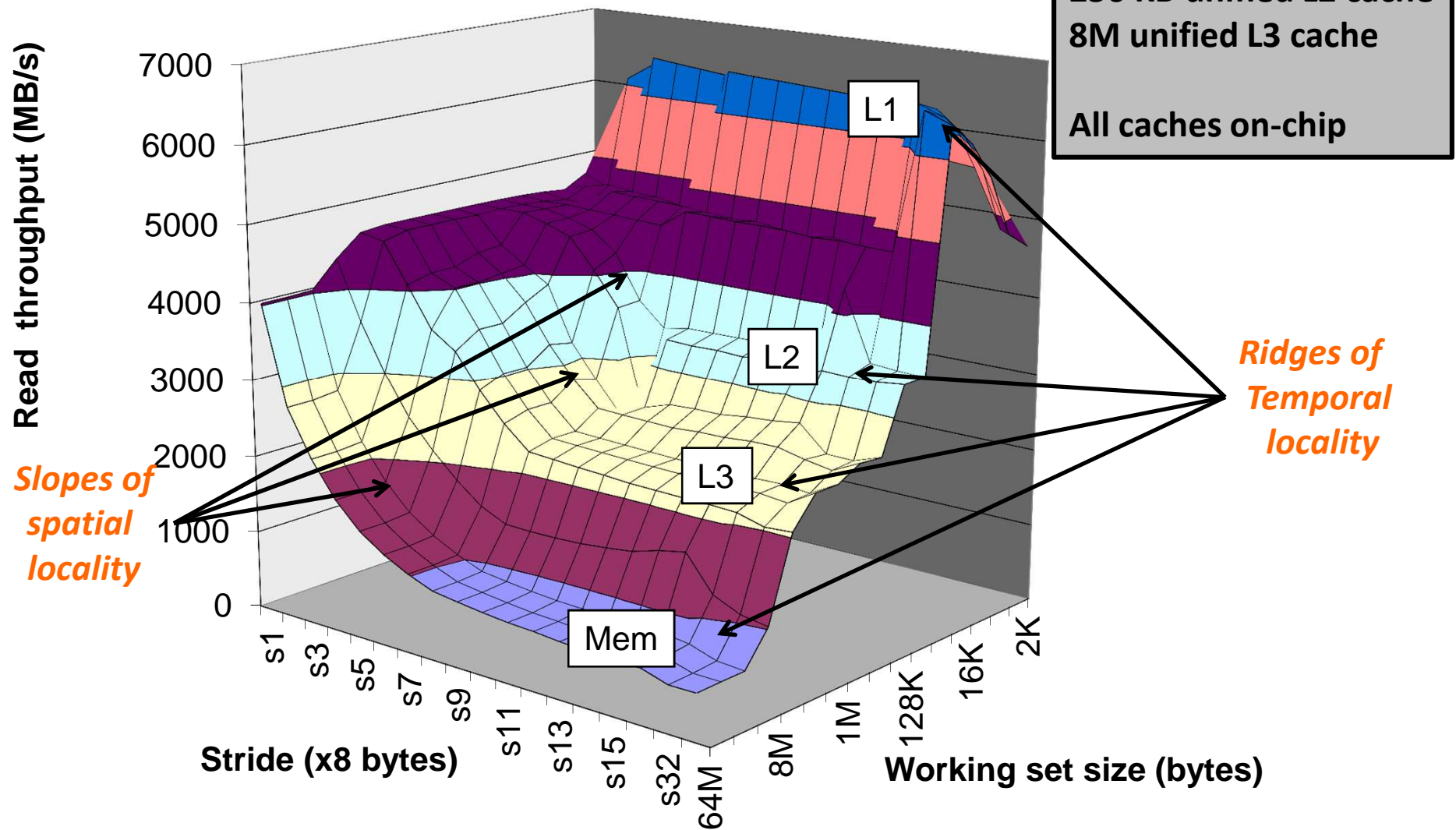
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The Memory Mountain



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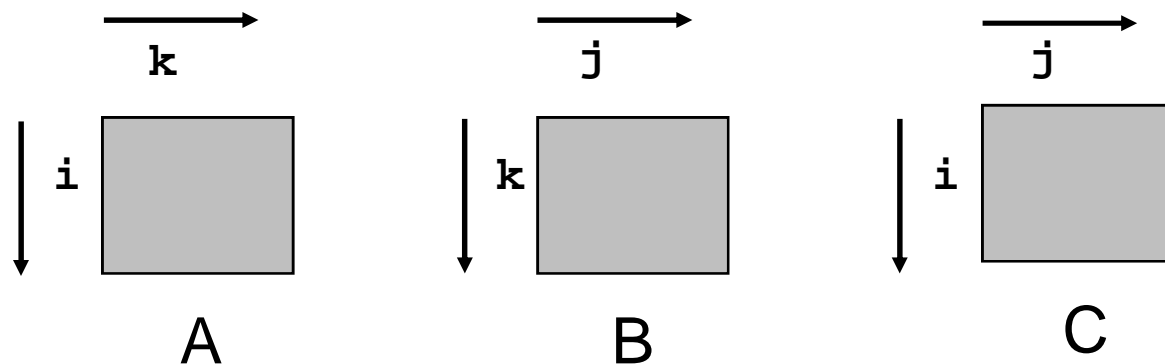
Miss Rate Analysis for Matrix Multiply

■ Assume:

- Line size = 32B (big enough for four 64-bit words)
- Matrix dimension (N) is very large
 - Approximate $1/N$ as 0.0
- Cache is not even big enough to hold multiple rows

■ Analysis Method:

- Look at access pattern of inner loop



Matrix Multiplication Example

■ Description:

- Multiply $N \times N$ matrices
- $O(N^3)$ total operations
- N reads per source element
- N values summed per destination
 - but may be able to hold in register

```
/* ijk */  
for (i=0; i<n; i++) {  
    for (j=0; j<n; j++) {  
        sum = 0.0;  
        for (k=0; k<n; k++)  
            sum += a[i][k] * b[k][j];  
        c[i][j] = sum;  
    }  
}
```

*Variable sum
held in register*

Layout of C Arrays in Memory (review)

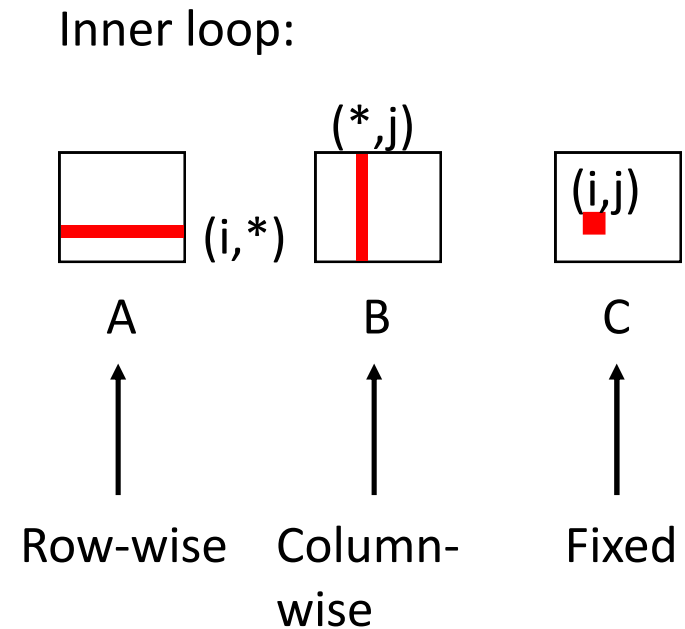
- **C arrays allocated in row-major order**
 - each row in contiguous memory locations
- **Stepping through columns in one row:**
 - ```
for (i = 0; i < N; i++)
 sum += a[0][i];
```
  - accesses successive elements
  - if block size (B) > 4 bytes, exploit spatial locality
    - compulsory miss rate = 4 bytes / B
- **Stepping through rows in one column:**
  - ```
for (i = 0; i < n; i++)  
    sum += a[i][0];
```
 - accesses distant elements
 - no spatial locality!
 - compulsory miss rate = 1 (i.e. 100%)

Matrix Multiplication (ijk)

```

/* ijk */
for (i=0; i<n; i++) {
  for (j=0; j<n; j++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum;
  }
}

```



Misses per inner loop iteration:

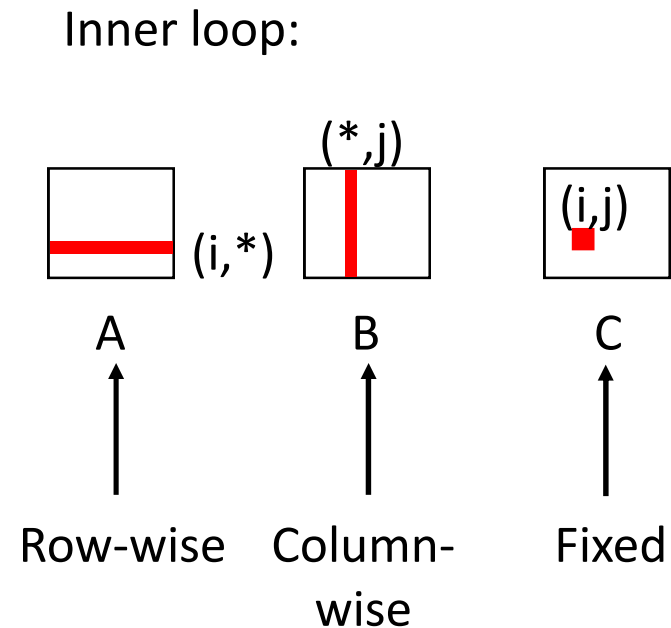
<u>A</u>	<u>B</u>	<u>C</u>
0.25	1.0	0.0

Matrix Multiplication (jik)

```

/* jik */
for (j=0; j<n; j++) {
  for (i=0; i<n; i++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum
  }
}

```



Misses per inner loop iteration:

<u>A</u>	<u>B</u>	<u>C</u>
0.25	1.0	0.0

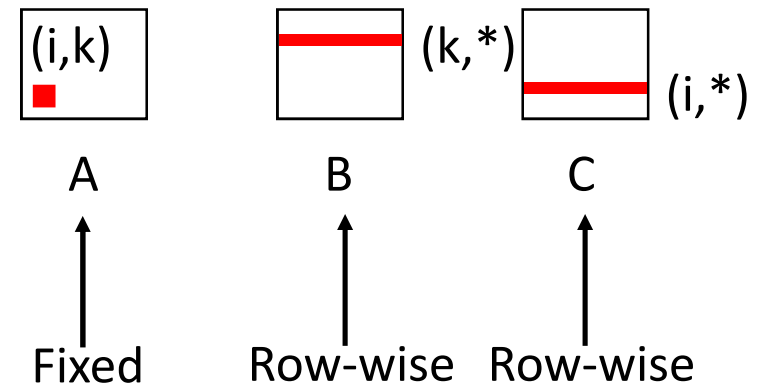
Matrix Multiplication (kij)

```

/* kij */
for (k=0; k<n; k++) {
  for (i=0; i<n; i++) {
    r = a[i][k];
    for (j=0; j<n; j++)
      c[i][j] += r * b[k][j];
  }
}

```

Inner loop:



Misses per inner loop iteration:

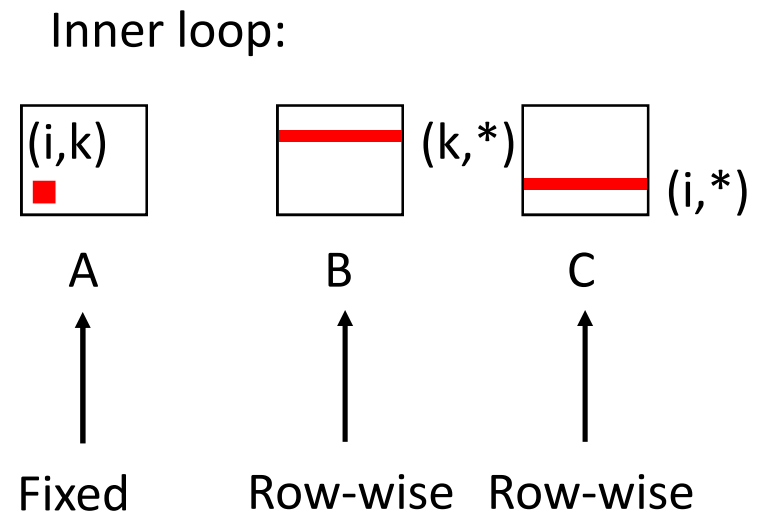
<u>A</u>	<u>B</u>	<u>C</u>
0.0	0.25	0.25

Matrix Multiplication (ikj)

```

/* ikj */
for (i=0; i<n; i++) {
    for (k=0; k<n; k++) {
        r = a[i][k];
        for (j=0; j<n; j++)
            c[i][j] += r * b[k][j];
    }
}

```



Misses per inner loop iteration:

<u>A</u>	<u>B</u>	<u>C</u>
0.0	0.25	0.25

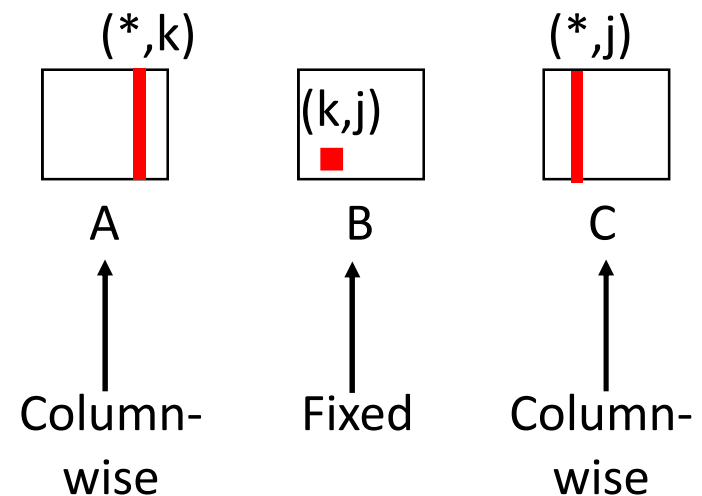
Matrix Multiplication (jki)

```

/* jki */
for (j=0; j<n; j++) {
  for (k=0; k<n; k++) {
    r = b[k][j];
    for (i=0; i<n; i++)
      c[i][j] += a[i][k] * r;
  }
}

```

Inner loop:



Misses per inner loop iteration:

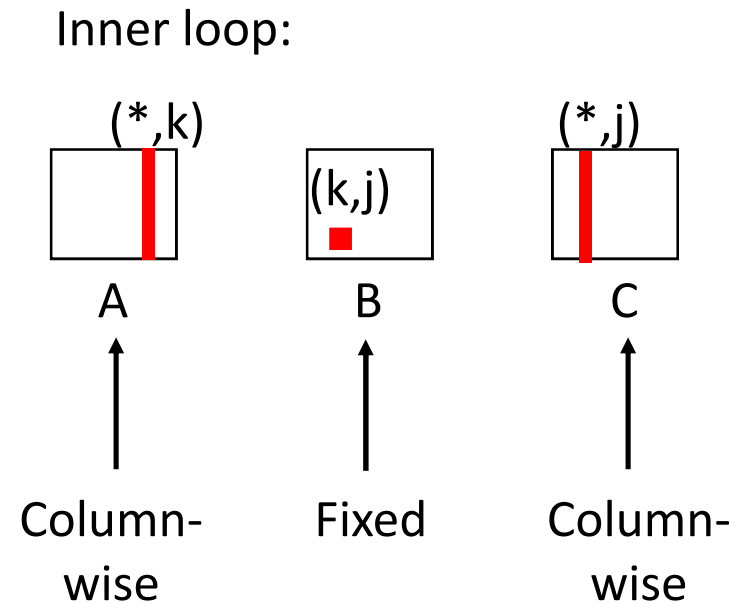
<u>A</u>	<u>B</u>	<u>C</u>
1.0	0.0	1.0

Matrix Multiplication (kji)

```

/* kji */
for (k=0; k<n; k++) {
    for (j=0; j<n; j++) {
        r = b[k][j];
        for (i=0; i<n; i++)
            c[i][j] += a[i][k] * r;
    }
}

```



Misses per inner loop iteration:

<u>A</u>	<u>B</u>	<u>C</u>
1.0	0.0	1.0

Summary of Matrix Multiplication

```
for (i=0; i<n; i++) {
    for (j=0; j<n; j++) {
        sum = 0.0;
        for (k=0; k<n; k++)
            sum += a[i][k] * b[k][j];
        c[i][j] = sum;
    }
}
```

ijk (& jik):

- 2 loads, 0 stores
- misses/iter = **1.25**

```
for (k=0; k<n; k++) {
    for (i=0; i<n; i++) {
        r = a[i][k];
        for (j=0; j<n; j++)
            c[i][j] += r * b[k][j];
    }
}
```

kij (& ikj):

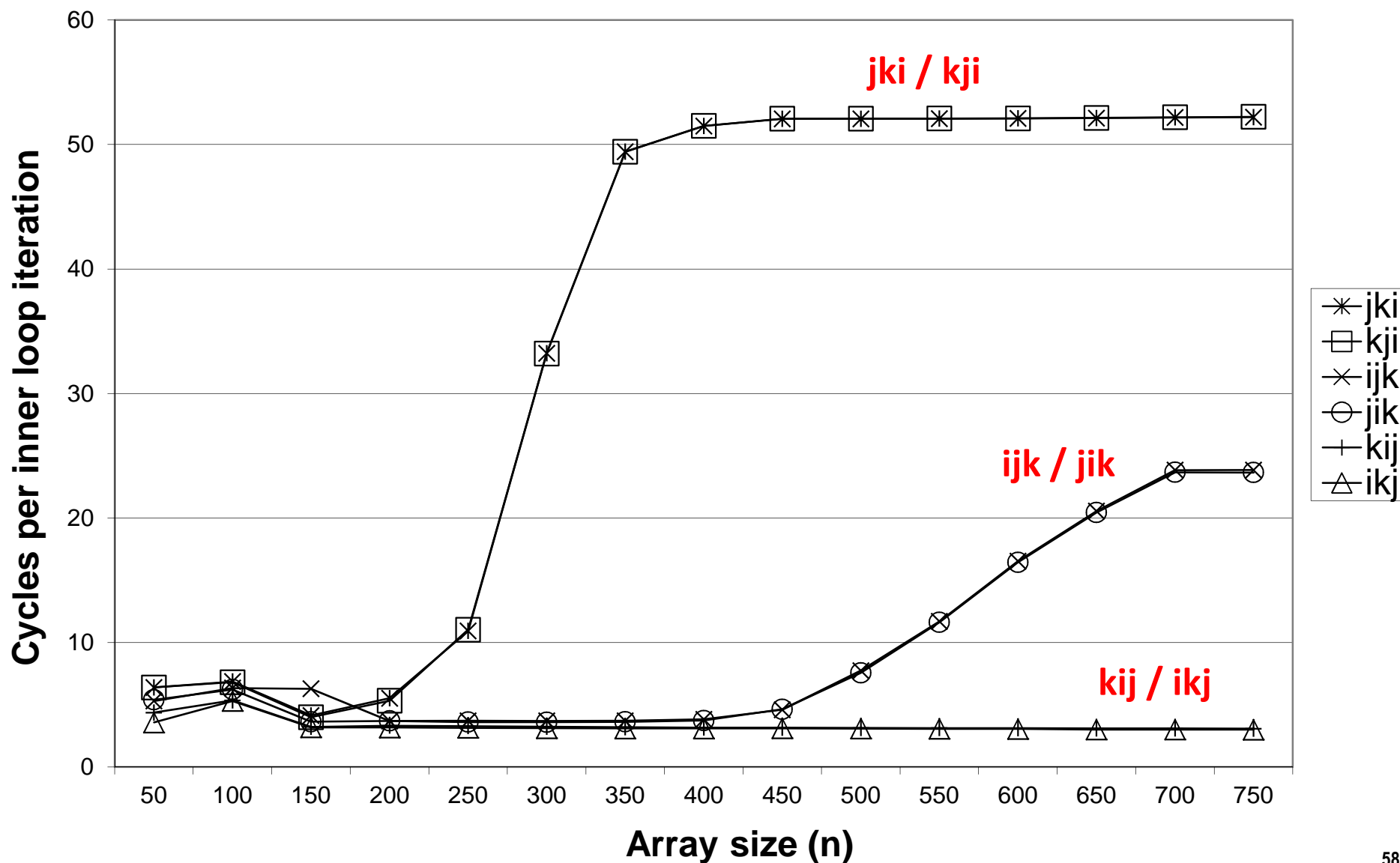
- 2 loads, 1 store
- misses/iter = **0.5**

```
for (j=0; j<n; j++) {
    for (k=0; k<n; k++) {
        r = b[k][j];
        for (i=0; i<n; i++)
            c[i][j] += a[i][k] * r;
    }
}
```

jki (& kji):

- 2 loads, 1 store
- misses/iter = **2.0**

Core i7 Matrix Multiply Performance



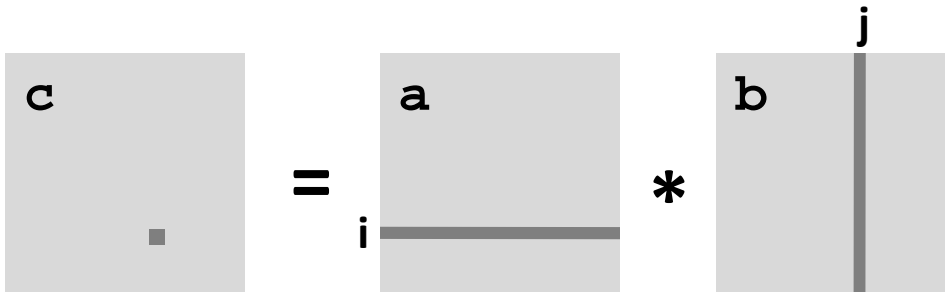
Today

- SRAM vs. DRAM
- Locality of reference
- Cache organization and operation
- Performance impact of caches
 - The memory mountain
 - Rearranging loops to improve spatial locality
 - Using blocking to improve temporal locality

Example: Matrix Multiplication

```
c = (double *) calloc(sizeof(double), n*n);

/* Multiply n x n matrices a and b */
void mmm(double *a, double *b, double *c, int n) {
    int i, j, k;
    for (i = 0; i < n; i++)
        for (j = 0; j < n; j++)
            for (k = 0; k < n; k++)
                c[i*n+j] += a[i*n + k]*b[k*n + j];
}
```



Cache Miss Analysis

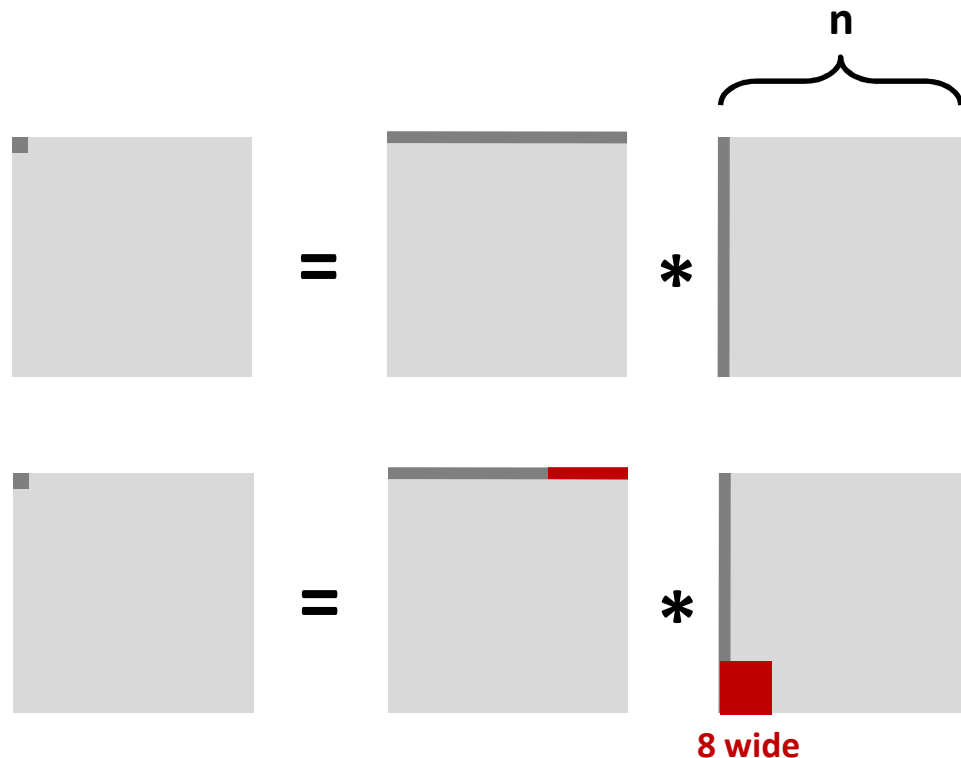
■ Assume:

- Matrix elements are doubles
- Cache block = 8 doubles
- Cache size $C \ll n$ (much smaller than n)

■ First iteration:

- $n/8 + n = 9n/8$ misses

- Afterwards **in cache:**
(schematic)



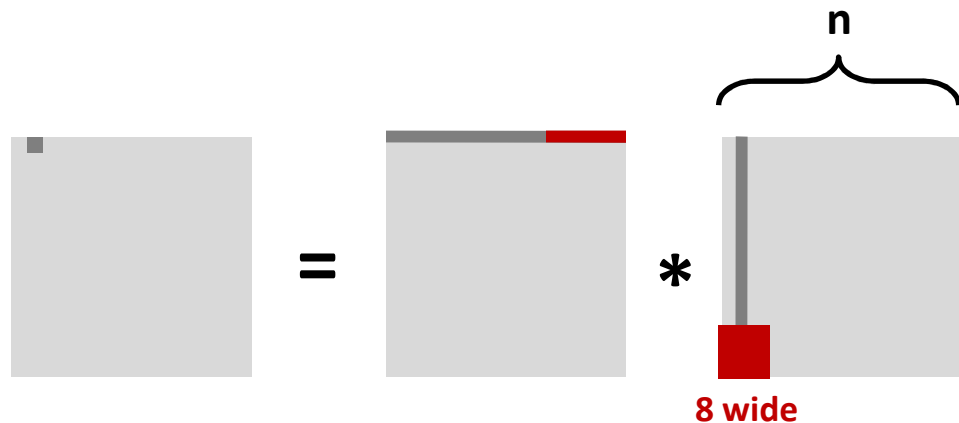
Cache Miss Analysis

■ Assume:

- Matrix elements are doubles
- Cache block = 8 doubles
- Cache size $C \ll n$ (much smaller than n)

■ Second iteration:

- Again:
 $n/8 + n = 9n/8$ misses



■ Total misses:

- $9n/8 * n^2 = (9/8) * n^3$

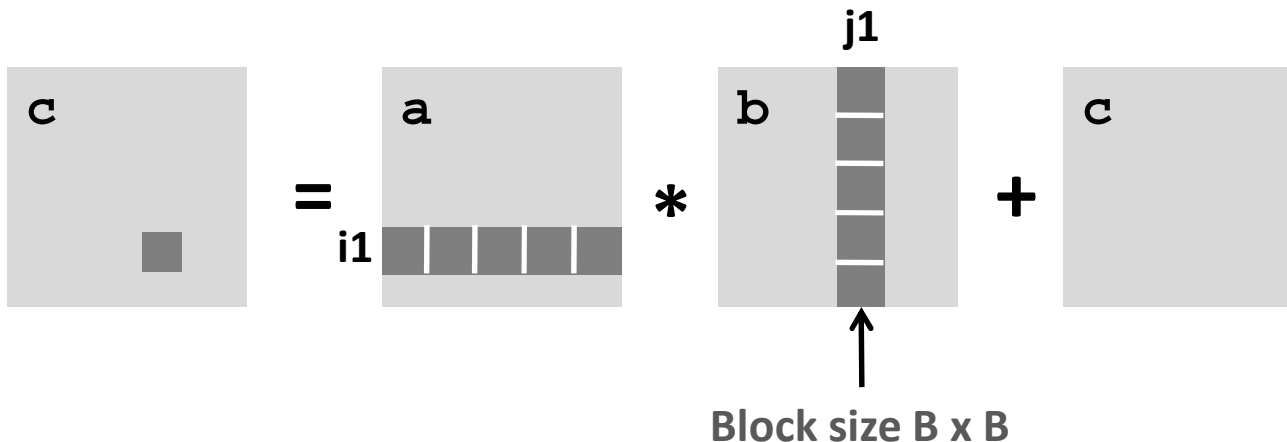
Blocked Matrix Multiplication

```

c = (double *) calloc(sizeof(double), n*n);


/* Multiply n x n matrices a and b */
void mmm(double *a, double *b, double *c, int n) {
    int i, j, k;
    for (i = 0; i < n; i+=B)
        for (j = 0; j < n; j+=B)
            for (k = 0; k < n; k+=B)
                /* B x B mini matrix multiplications */
                for (i1 = i; i1 < i+B; i++)
                    for (j1 = j; j1 < j+B; j++)
                        for (k1 = k; k1 < k+B; k++)
                            c[i1*n+j1] += a[i1*n + k1]*b[k1*n + j1];
}

```



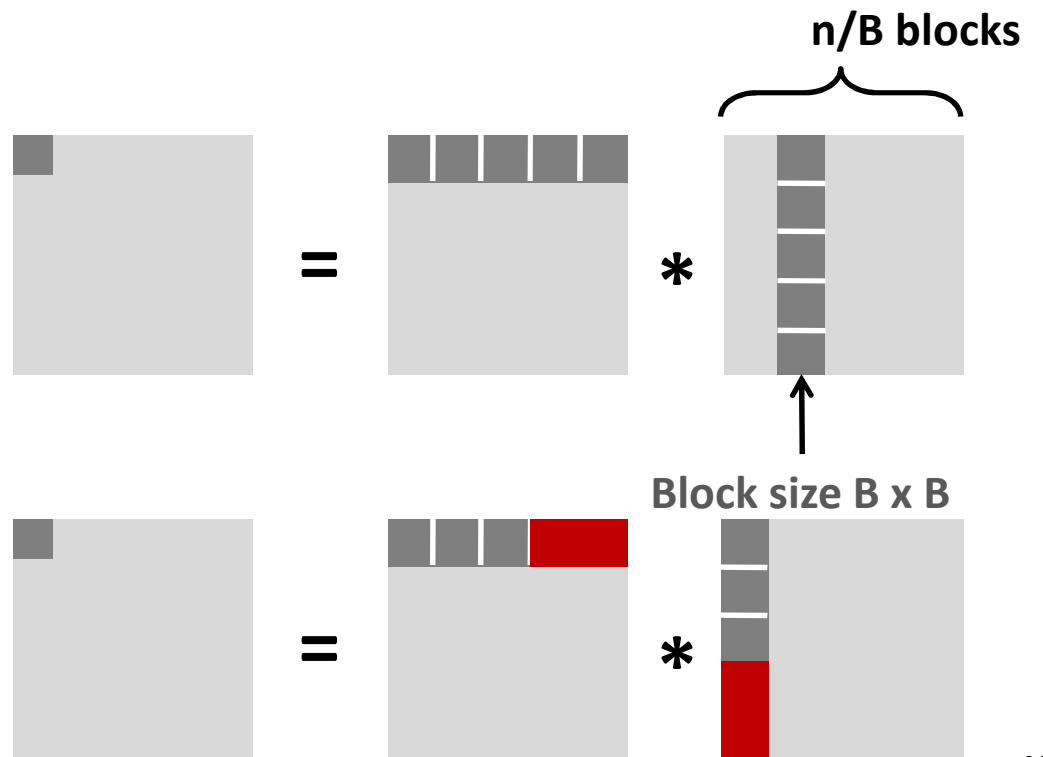
Cache Miss Analysis

■ Assume:

- Cache block = 8 doubles
- Cache size $C \ll n$ (much smaller than n)
- Three blocks  fit into cache: $3B^2 < C$


■ First (block) iteration:

- $B^2/8$ misses for each block
- $2n/B * B^2/8 = nB/4$
(omitting matrix c)



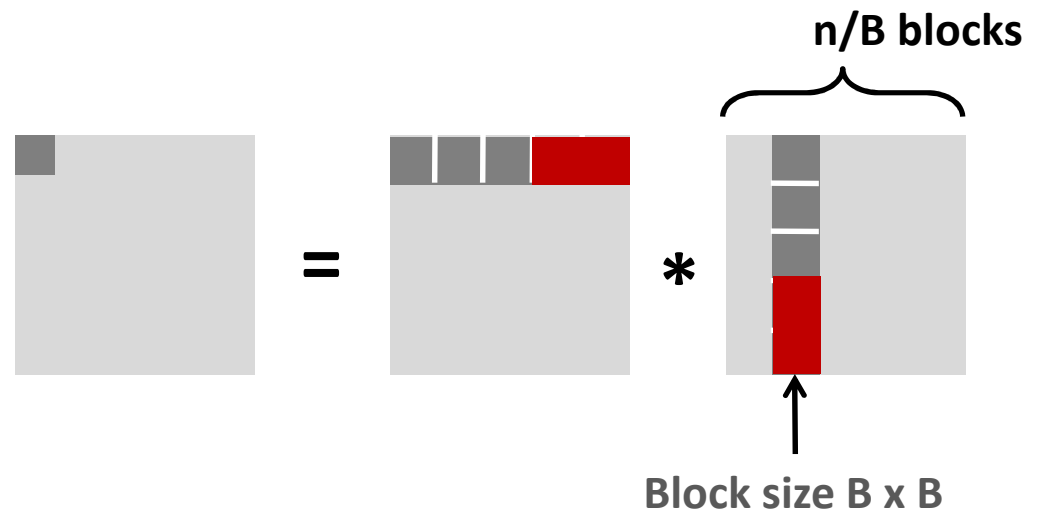
Cache Miss Analysis

■ Assume:

- Cache block = 8 doubles
- Cache size $C \ll n$ (much smaller than n)
- Three blocks  fit into cache: $3B^2 < C$

■ Second (block) iteration:

- Same as first iteration
- $2n/B * B^2/8 = nB/4$



■ Total misses:

- $nB/4 * (n/B)^2 = n^3/(4B)$

Summary

- No blocking: $(9/8) * n^3$
- Blocking: $1/(4B) * n^3$
- Suggest largest possible block size B , but limit $3B^2 < C!$
- Reason for dramatic difference:
 - Matrix multiplication has inherent temporal locality:
 - Input data: $3n^2$, computation $2n^3$
 - Every array elements used $O(n)$ times!
 - But program has to be written properly

Concluding Observations

- **Programmer can optimize for cache performance**
 - How data structures are organized
 - How data are accessed
 - Nested loop structure
 - Blocking is a general technique
- **All systems favor “cache friendly code”**
 - Getting absolute optimum performance is very platform specific
 - Cache sizes, line sizes, associativities, etc.
 - Can get most of the advantage with generic code
 - Keep working set reasonably small (temporal locality)
 - Use small strides (spatial locality)