Data Representation in Memory

CSCI 224 / ECE 317: Computer Architecture

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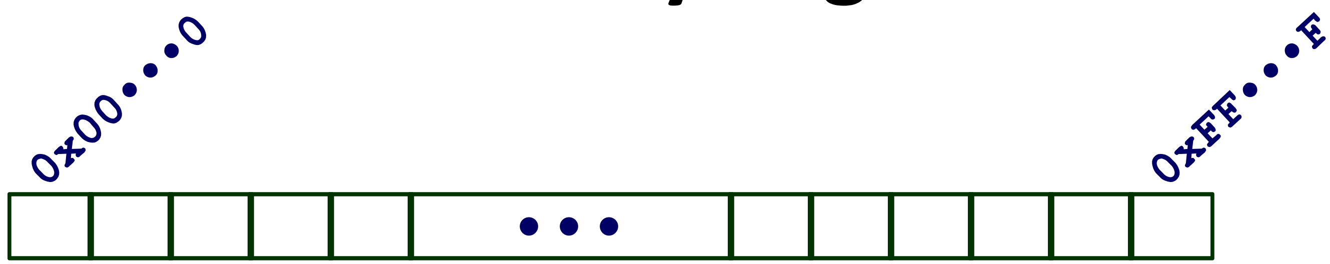
Slides adapted from Bryant & O'Hallaron's slides

Data Representation in Memory

- **Basic memory organization**
- Bits & Bytes – basic units of Storage in computers
- Representing information in binary and hexadecimal
- Representing Integers
  - Unsigned integers
  - Signed integers
- Representing Text
- Representing Pointers

Byte-Oriented Memory Organization



- **Byte-Addressable Memory**
  - Conceptually a very large array of bytes
  - Each byte has a unique address
  - Processor width determines address range:
    - 32-bit processor has 2^{32} unique addresses: 4GB max
      - 0x00000000 to 0xffffffff
    - 64-bit processor has 2^{64} unique addresses: ~ 1.8×10^{19} bytes max
      - 0x0000000000000000 to 0xffffffffffffffff
  - Memory implemented with hierarchy of different memory types
  - OS provides virtual address space private to particular "process"
    - virtual memory to be discussed later…

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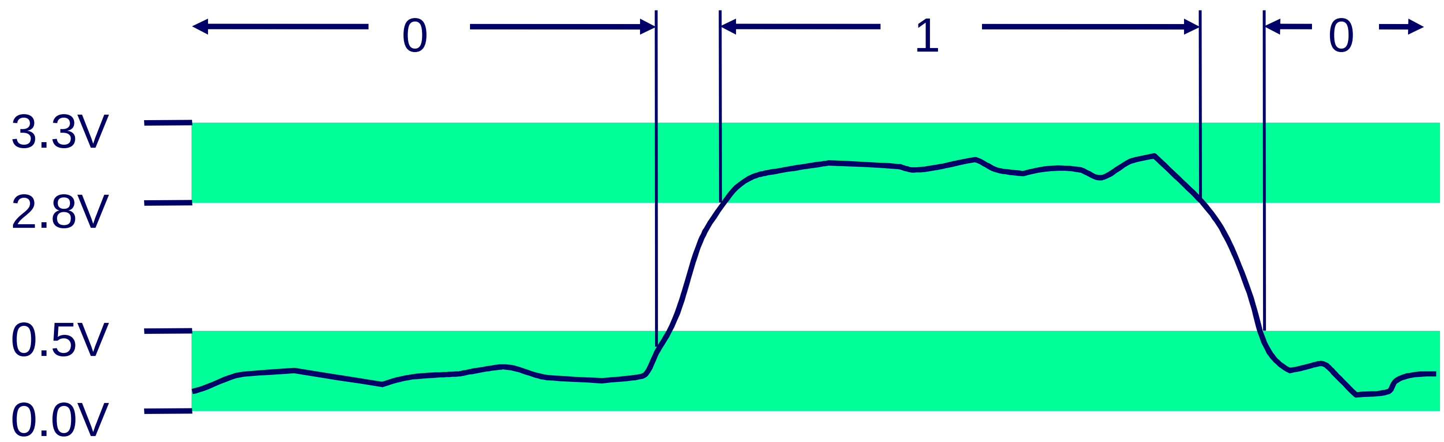
Representing Voltage Levels as Binary Values



- Digital transistors operate in high and low voltage ranges
- Voltage Range dictates Binary Value on wire
  - high voltage range (e.g. 2.8V to 3.3V) is a logic 1
  - low voltage range (e.g. 0.0V to 0.5V) is a logic 0
  - voltages in between are indefinite values

Bits & Bytes

- **Transistors have two states, so computers use bits**
  - "bit" is a base-2 digit
  - {L, H} => {0, 1}
- **Single bit offers limited range, so grouped in bytes**
  - 1 byte = 8 bits
  - a single datum may use multiple bytes
- **Data representation 101:**
  - Given N bits, can represent 2^N unique values

Encoding Byte Values

- **Processors generally use multiples of Bytes**
  - common sizes: 1, 2, 4, 8, or 16 bytes
  - Intel data names:

Byte	1 byte	(8 bits)
Word	2 bytes	(16 bits)
Double word	4 bytes	(32 bits)
Quad word	8 bytes	(64 bits)

Unfortunately, most processor architectures call 2 bytes a 'halfword', 4 bytes a 'word', etc., so we'll often use C data names instead (but these vary in size too… /sigh)

C Data Types

C Data Type	Typical 32-bit	Intel IA32 *32-bit*	x86-64 *64-bit*
char	1	1	1
short	2	2	2
int	4	4	4
long	4	***4***	***8***
long long	8	8	8
float	4	4	4
double	8	8	8
long double	8	10/12	10/16
pointer (addr)	4	***4***	***8***

key differences

Data Representation in Memory

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Encoding Byte Values

- **1 Byte = 8 bits**
  - Binary: 00000000_2 to 11111111_2
- **A byte value can be interpreted in many ways!**
  - depends upon how it's used
- **For example, consider byte with: 01010101_2**
  - as text: 'U'
  - as integer: 85_{10}
  - as IA32 instruction: pushl %ebp
  - part of an address or real number
  - a medium gray pixel in a gray-scale image
  - could be interpreted MANY other ways…

Encoding Byte Values

- **Different syntaxes for a byte**
  - Binary: 00000000_2 to 11111111_2
  - Decimal: 0_{10} to 255_{10}
  - Hexadecimal: 00_{16} to FF_{16}
    - Base-16 number representation
    - Use characters '0' to '9' and 'A' to 'F'
    - in C/C++ programming languages, $D3_{16}$ written as either
      - 0xD3
      - 0xd3

Decimal vs Binary vs Hexadecimal

Decimal	*Binary*	*Hexadecimal*
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7
8	1000	8
9	1001	9
10	1010	A
11	1011	B
12	1100	C
13	1101	D
14	1110	E
15	1111	F

Binary and Hexadecimal Equivalence

- **Problem with binary – Hard to gauge size of binary numbers**
  - e.g. approx. how big is: $1010011101010001011101011_2$?
  - Would be nice if native computer base was decimal: 21,930,731
    - but a decimal digit requires 3.322 bits… *won't work*

- **Need a larger base equivalent to binary, such that**

$$\boldsymbol{R^1 = 2^x}$$

  - for equivalence, *R* and *x* must be integers – then 1 digit in *R* equals *x* bits
  - equivalence allows direct conversion between representations
  - two options closest to decimal:
    - octal: $\mathbf{8^1 = 2^3}$
    - hexadecimal: $\mathbf{16^1 = 2^4}$

Binary and Hexadecimal Equivalence

- **Octal or Hexadecimal?**
  - binary : $1010011101010001011101011_2$
  - octal: 123521353_8
  - hexadecimal number: $14EA2EB_{16}$
  - decimal: 21930731
- **Octal a little closer in size to decimal, BUT…**
- **How many base-*R* digits per byte?**
  - Octal: 8/3 = 2.67 octal digits per byte -- BAD
  - Hex: 8/4 = 2 hex digits per byte -- GOOD

Hexadecimal wins: 1 hex digit ⇔ 4 bits

Convert Between Binary and Hex

Convert Hexadecimal to Binary

- Simply replace each hex digit with its equivalent 4-bit binary sequence
- Example: $6\,D\,1\,9\,F\,3\,C_{16}$

$$\mathbf{0110 \quad 1101 \quad 0001 \quad 1001 \quad 1111 \quad 0011 \quad 1100_2}$$

Convert Binary to Hexadecimal

- <u>Starting from the radix point</u>, replace each sequence of 4 bits with the equivalent hexadecimal digit
- Example: $101100100011010111010110001010011_2$

$$\mathbf{1 \quad 6 \quad 4 \quad 6 \quad B \quad A \quad C \quad 5 \quad 3_{16}}$$

Data Representation in Memory

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  - **Unsigned integers**
  - Signed integers
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Unsigned Integers – Binary

- **Computers store Unsigned Integer numbers in Binary (base-2)**
  - Binary numbers use place valuation notation, just like decimal
  - Decimal value of *n*-bit unsigned binary number:

$$value_{10} = \sum_{i=0}^{n-1} a_i * 2^i$$

0	1	1	1	0	1	0	1
2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0

$$\begin{aligned} value_{10} &= 0 * 2^7 + 1 * 2^6 + 1 * 2^5 + 1 * 2^4 + 0 * 2^3 + 1 * 2^2 + 0 * 2^1 + 1 * 2^0 \\ &= 2^6 + 2^5 + 2^4 + 2^2 + 2^0 \\ &= 64 + 32 + 16 + 4 + 1 \boxed{= 117_{10}} \end{aligned}$$

Unsigned Integers – Base-R

- **Convert Base-R to Decimal**
  - Place value notation can similarly determine decimal value of any base, *R*
  - Decimal value of *n*-digit base *r* number:

$$value_{10} = \sum_{i=0}^{n-1} a_i * r^i$$

  - Example: $317_8 = ?_{10}$

$$\begin{aligned} value_{10} &= 3 * 8^2 + 1 * 8^1 + 7 * 8^0 \\ &= 3 * 64 + 1 * 8 + 7 * 1 \\ &= 192 + 8 + 7 \; \boxed{= 207_{10}} \end{aligned}$$

Unsigned Integers – Hexadecimal

- **Commonly used for converting hexadecimal numbers**
  - Hexadecimal number is an "equivalent" representation to binary, so often need to determine decimal value of a hex number
  - Decimal value for *n*-digit hexadecimal (base 16*)* number:

$$value_{10} = \sum_{i=0}^{n-1} a_i * 16^i$$

  - Example: $9E4_{16} = ?_{10}$

$$value_{10} = 9 * 16^2 + 14 * 16^1 + 4 * 16^0$$
$$= 9 * 256 + 14 * 16 + 4 * 1$$
$$= 2304 + 224 + 4 \boxed{= 2532_{10}}$$

Unsigned Integers – Convert Decimal to Base-R

- **Also need to convert decimal numbers to desired base**
- **Algorithm for converting unsigned Decimal to Base-R**
  a) Assign decimal number to *NUM*
  b) Divide *NUM* by *R*
     - Save remainder *REM* as next least significant digit
     - Assign quotient *Q* as new *NUM*
  c) Repeat step b) until quotient *Q* is zero
- **Example:** $\mathbf{83_{10} = ?_7}$

least significant digit

NUM		*R*		*Q*		*REM*
$\mathbf{83}$	/	$\mathbf{7}$	$\rightarrow$	$\mathbf{11}$	$\boldsymbol{r}$	$\mathbf{6}$
$\mathbf{11}$	/	$\mathbf{7}$	$\rightarrow$	$\mathbf{1}$	$\boldsymbol{r}$	$\mathbf{4}$
$\mathbf{1}$	/	$\mathbf{7}$	$\rightarrow$	$\mathbf{0}$	$\boldsymbol{r}$	$\mathbf{1}$

$$\boxed{\mathbf{= 146_7}}$$

most significant digit

Unsigned Integers – Convert Decimal to Binary

- **Example with Unsigned Binary:** $52_{10} = ?_2$

least significant digit

NUM		*R*		*Q*		*REM*
52	$/$	2	$\rightarrow$	26	r	0
26	$/$	2	$\rightarrow$	13	r	0
13	$/$	2	$\rightarrow$	6	r	1
6	$/$	2	$\rightarrow$	3	r	0
3	$/$	2	$\rightarrow$	1	r	1
1	$/$	2	$\rightarrow$	0	r	1

$$\boxed{= 110100_2}$$

most significant digit

Unsigned Integers – Convert Decimal to Hexadecimal

- **Example with Unsigned Hexadecimal:** $437_{10} = ?_{16}$

NUM *R* *Q* *REM*

$$437 \;/\; 16 \rightarrow 27 \quad r \quad 5$$
$$27 \;/\; 16 \rightarrow 1 \quad r \quad 11$$
$$1 \;/\; 16 \rightarrow 0 \quad r \quad 1$$

least significant digit

$$\boxed{= 1B5_{16}}$$

most significant digit

Unsigned Integers – Ranges

- **Range of Unsigned binary numbers based on number of bits**
  - Given representation with *n* bits, min value is always sequence
    - 0....0000 = 0
  - Given representation with *n* bits, max value is always sequence
    - 1....1111 = $2^n - 1$
  - So, ranges are:
    - unsigned char: $\mathbf{0 \rightarrow 255 \quad (2^8 - 1)}$
    - unsigned short: $\mathbf{0 \rightarrow 65,535 \quad (2^{16} - 1)}$
    - unsigned int: $\mathbf{0 \rightarrow 4,294,967,295 \quad (2^{32} - 1)}$

1	1	. . .	1	1	1	1
2^{n-1}	2^{n-2}		2^3	2^2	2^1	2^0

$$= \sum_{i=0}^{n-1} 2^i = 2^n - 1$$

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Signed Integers – Binary

- **Signed Binary Integers converts half of range as negative**
- **Signed representation identical, except for most significant bit**
  - For signed binary, most significant bit indicates sign
    - 0 for nonnegative
    - 1 for negative
  - <u>Must know number of bits</u> for signed representation

Place value of most significant bit is <u>negative</u> for signed binary

***Unsigned* Integer representation:**

2^7 2^6 2^5 2^4 2^3 2^2 2^1 2^0

***Signed* Integer representation:**

-2^7 2^6 2^5 2^4 2^3 2^2 2^1 2^0

Signed Integers – Binary

- **Decimal value of *n*-bit signed binary number:**

$$value_{10} = -a_{n-1} * 2^{n-1} + \sum_{i=0}^{n-2} a_i * 2^i$$

- **Positive (in-range) numbers have same representation:**

***Unsigned* Integer representation:**

0	1	1	0	1	0	0	1
2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0

$= 105_{10}$

***Signed* Integer representation:**

0	1	1	0	1	0	0	1
-2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0

$= 105_{10}$

Signed Integers – Binary

- Only when most significant bit set does value change
- Difference between unsigned and signed integer values is 2^N

Unsigned **Integer representation:**

~~0~~ 1	1	1	0	1	0	0	1
2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0

$$= \mathbf{105}$$
$$\boxed{+\,\mathbf{128}}\mathbf{10}$$
$$= \mathbf{233_{10}}$$

Signed **Integer representation:**

~~0~~ 1	1	1	0	1	0	0	1
-2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0

$$= \mathbf{105}$$
$$\boxed{-\,\mathbf{128}}\mathbf{10}$$
$$= \mathbf{-23_{10}}$$

Signed Integers – Ranges

- **Range of Signed binary numbers:**
  - Given representation with *n* bits, min value is always sequence
    - 100....0000 = -2^{n-1}
  - Given representation with *n* bits, max value is always sequence
    - 011....1111 = $2^{n-1} - 1$
  - So, ranges are:

C data type	# bits	Unsigned range	Signed range
char	8	0 → 255	-128 → 127
short	16	0 → 65,535	-32,768 → 32,767
int	32	0 → 4,294,967,295	-2,147,483,648 → 2,147,483,647

Signed Integers – Convert to/from Decimal

- **Convert Signed Binary Integer to Decimal**
  - Easy – just use place value notation
    - two examples given on last two slides
- **Convert Decimal to Signed Binary Integer**
  - MUST know <u>number of bits</u> in signed representation
  - ***Algorithm***:
    a) Convert magnitude (abs val) of decimal number to unsigned binary
    b) Decimal number originally negative?
      – If positive, conversion is <u>done</u>
      – If negative, perform negation on answer from part a)
        » zero extend answer from a) to N bits (size of signed repr)
        » negate: flip bits and add 1

Signed Integers – Convert Decimal to Base-R

- **Example:** $-37_{10} = ?_{8-bit\ signed}$
  - A) $|-37_{10}| = ?_2$

NUM	*R*		*Q*		*REM*
37	$/\ 2$	$\rightarrow$	18	r	1
18	$/\ 2$	$\rightarrow$	9	r	0
9	$/\ 2$	$\rightarrow$	4	r	1
4	$/\ 2$	$\rightarrow$	2	r	0
2	$/\ 2$	$\rightarrow$	1	r	0
1	$/\ 2$	$\rightarrow$	0	r	1

least significant bit

$= 100101_2$

most significant bit

Signed Integers – Convert Decimal to Base-R

- **Example:** $-37_{10} = ?_{8-bit\ signed}$
  - B) -37_{10} was negative, so perform *negation*
    - zero extend 100101 to 8 bits

$$\mathbf{100101_2} \rightarrow \mathbf{\underline{00}100101_2}$$

    - negation
      - flip bits: $\mathbf{00100101_2}$

$$\downarrow$$

$$\mathbf{11011010_2}$$

      - add 1:

$$\begin{array}{r} \mathbf{11011010_2} \\ + \quad \mathbf{1_2} \\ \hline \mathbf{11011011_2} \end{array}$$

$$\boxed{= \mathbf{11011011_2}}$$

Can validate answer using place value notation

Signed Integers – Convert Decimal to Base-R

- **Example:** $67_{10} = ?_{8-bit\ signed}$
  - A) $|67_{10}| = ?_2$

NUM		R		Q		REM
67	/	2	$\rightarrow$	33	r	1
33	/	2	$\rightarrow$	16	r	1
16	/	2	$\rightarrow$	8	r	0
8	/	2	$\rightarrow$	4	r	0
4	/	2	$\rightarrow$	2	r	0
2	/	2	$\rightarrow$	1	r	0
1	/	2	$\rightarrow$	0	r	1

least significant bit

$$\boxed{= 1000011_2}$$

most significant bit

Signed Integers – Convert Decimal to Base-R

- **Example:** $67_{10} = ?_{8-bit\ signed}$
  - B) 67_{10} was positive, so <u>done</u>

$$\boxed{= 1000011_2}$$

Can validate answer using place value notation

Signed Integers – Convert Decimal to Base-R

- **Be careful of range!**
- **Example:** $-\mathbf{183_{10}} = ?_{\boldsymbol{8-bit\ signed}}$
  - A) $|-\mathbf{183_{10}}| = ?_{\mathbf{2}}$ $= \mathbf{10110111_2}$
  - B) -183_{10} was negative, so perform *negation*
    - zero extend 10110111 to 8 bits // already done
    - negation
      - flip bits: $\mathbf{10110111_2}$

$$\downarrow$$

$$\mathbf{01001000_2}$$

      - add 1: $+ \quad \mathbf{1_2}$

$$\mathbf{01001001_2} = \mathbf{73_{10}}$$

<u>not</u> -183_{10}… WRONG!

-183_{10} is not in valid range for 8-bit signed

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Representing Strings

```
char S[6] = "18243";
```

- **Strings in C**
  - Represented by array of characters
  - Each character encoded in **ASCII format**
    - Standard 7-bit encoding of character set
    - Character “0” has code 0x30
  - String should be null-terminated
    - Final character = 0
  - ASCII characters organized such that:
    - Numeric characters sequentially increase from 0x30
      - Digit *i* has code 0x30+*i*
    - Alphabetic characters sequentially increase in order
      - Uppercase chars ‘A’ to ‘Z’ are 0x41 to 0x5A
      - Lowercase chars ‘A’ to ‘Z’ are 0x61 to 0x7A
    - Control characters, like <RET>, <TAB>, <BKSPC>, are 0x00 to 0x1A

Intel / Linux

0x31	**‘1’**
0x38	**‘8’**
0x32	**‘2’**
0x34	**‘4’**
0x33	**‘3’**
0x00	**null term**

Representing Strings

Limitations of ASCII

- 7-bit encoding limits set of characters to $2^7 = 128$
- 8-bit extended ASCII exists, but still only $2^8 = 256$ chars
- Unable to represent most other languages in ASCII

Answer: *Unicode*

- first 128 characters are ASCII
  - i.e. 2-byte Unicode for '4': 0x34 -> 0x0034
  - i.e. 4-byte Unicode for 'T': 0x54 -> 0x00000054
- UTF-8: 1-byte version // commonly used
- UTF-16: 2-byte version // commonly used
  - allows 2^{16} = 65,536 unique chars
- UTF-32: 4-byte version
  - allows 2^{32} = ~4 billion unique characters
- Unicode used in many more recent languages, like Java and Python

UTF-16 on Intel

Byte	Character
0x31	'1'
0x00	
0x38	'8'
0x00	
0x32	'2'
0x00	
0x34	'4'
0x00	
0x33	'3'
0x00	
0x00	null term
0x00	

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Representing Pointers

```
int B = -15213;
int *P = &B;
```

Sun	IA32	x86-64
EF	D4	0C
FF	F8	89
FB	FF	EC
2C	BF	FF
		FF
		7F
		00
		00

Different compilers & machines assign different locations to objects