

Homework #2: Basics of Algorithm Analysis
Due Date: Friday, 3 February 2006

Guidelines

Please make sure you adhere to the policies on collaboration and academic honesty as outlined in syllabus.

Reading

Read Ch. 2 of text.

Practice

See Solved Exercises of Chapter 2 of text.

These exercises are purely for your own practice. You should not turn them in, and you are free to discuss them fully with others.

Problems

Problem A (6 points)

Exercise 2 of Chapter 2 of text (page 67)

Problem B (10 points)

Rank the following functions by order of growth, *i.e.*, find an arrangement g_1, g_2, \dots, g_{20} of the functions satisfying $g_1 = \Omega(g_2), g_2 = \Omega(g_3), \dots, g_{19} = \Omega(g_{20})$. Partition your list into equivalence classes such that $f(n)$ and $g(n)$ are in the same class iff $f(n) = \Theta(g(n))$.

$$\begin{array}{cccccc} n & 2^{2^n} & (\sqrt{2})^{\lg n} & n^2 & n! & \\ (3/2)^n & n^3 & n \lg n & \lg(n!) & \ln \ln n & \\ n \cdot 2^n & n^{\lg \lg n} & \ln n & 2^{\lg n} & (\lg n)^{\lg n} & \\ e^n & 4^{\lg n} & 2^n & 1 & \sqrt{\lg n} & \end{array}$$

[Note: We do not require formal proof for this problem, simply the ordering and the equivalence classes.]

Problem C (16 points)

Let $f(n)$ and $g(n)$ be functions such that $f(n) = \Omega(1)$ and $g(n) = \Omega(1)$. Prove or disprove each of the following conjectures.

- i. $f(n) = O(g(n))$ implies $g(n) = O(f(n))$.
- ii. $f(n) + g(n) = \Theta(\min(f(n), g(n)))$.
- iii. $f(n) = O(g(n))$ implies $\lg(f(n)) = O(\lg(g(n)))$, where $\lg(g(n)) > 0$ and $f(n) \geq 1$ for all sufficiently large n .
- iv. $f(n) = O(g(n))$ implies $2^{f(n)} = O(2^{g(n)})$.
- v. $f(n) = O((f(n))^2)$.
- vi. $f(n) = O(g(n))$ implies $g(n) = \Omega(f(n))$.
- vii. $f(n) = \Theta(f(n/2))$.
- viii. $f(n) + g(n) = \Theta(f(n))$, where $g(n) = O(f(n))$.

[Your proofs must be formal, although for false conjectures a specific counterexample constitutes a valid proof.]

Problem D (8 points)

Exercise 6 of Chapter 2 of text (pages 68–69)

Problem E (**EXTRA CREDIT – 4 points**)

Exercise 8 of Chapter 2 of text (pages 69–70)