CSCI 314 – Spring 2008 Handout: asgn05
Algorithms
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Homework #5: Greedy Algorithms
Due Date: Friday, 4 April 2008

Problems

Problem A (10 points)

Our visitor posed the following problem. We are given chemicals $C_1, C_2, \ldots, C_n$, each of which has a safe temperature range $[a_i, b_i]$ at which it can be stored. The goal is to pick the smallest cardinality set of temperatures (a.k.a. "refrigerators") such that each chemical can be safely stored. The optimal algorithm that he outlined was based on picking the minimal b_i value as the first temperature, storing all chemicals that are safe to be stored at that temperature, and then repeating the process with the remaining chemicals.

For this homework, we want you to consider the following (flawed) algorithm design. Determine a temperature t that can be used to store the greatest number of chemicals. Then repeat this process, choosing another temperature that satisfies the greatest number of *remaining* chemicals, and so on. Demonstrate the inferiority of the plan as follows:

- Specify a set of temperature ranges as a problem instance.
- Give the optimal solution that results from the original (correct) algorithm.
- Give a non-optimal solution that results from the second (flawed) algorithm.

Problem B (10 points)

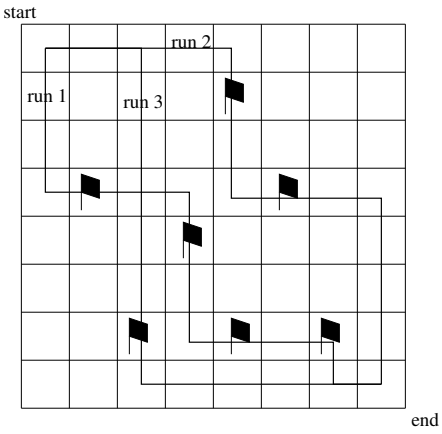
Consider the following optimization problem. Someone is in charge of collecting all of the flags left interspersed on the ski slope. However they must do so by skiing by and grabbing them. Given that they cannot ski uphill, it may actually take more than one run to gather all the flags. We would like to be able to suggest a plan that uses as few runs as possible.

We model the problem as follows. The slope itself will be viewed as an $n \times n$ grid. The top of the ski-slope will be the top-left corner of the grid, and the hill is sloped so that a skier can move either downward or rightward at each individual step, until eventually ending up at the bottom-right corner of the grid. The flags will be placed at certain grid locations and we will be informed of the overall number of flags and the precise grid coordinates of the flags before we begin.

Someone has suggested the following greedy approach: When planning the first run, calculate a path that maximizes the number of flags which will be collected

on that run (let's not worry about precisely how we calculate this local solution – just assume that we have a way to find such a path and use it). With those flags removed, repeat this approach, choosing a second run that maximizes the number of *remaining* flags that can be collected, and so on until collecting all flags.

An example of this approach on an instance is diagramed as follows.



Unfortunately, this algorithm does not always achieve a solution with the fewest number of overall runs. Demonstrate the inferiority of the plan as follows:

- Give an explicit instance of the problem, specifying the size of the grid and the exact placements of the flags within that grid.
- Clearly diagram the runs which you suggest would be chosen according the the greedy algorithm outlined.
- In a separate diagram, clearly identify a strictly smaller collection of runs which could have been used as a solution.

Problem C (10 points)

Exercise 2 of Chapter 4 of the text.

Problem D (10 points)

Exercise 13 of Chapter 4 of the text. Make sure to explain the correctness of your approach.

Problem E **(EXTRA CREDIT – 4 points)**

Describe an efficient algorithm that correctly solves the skiing cleanup problem. Clearly explain the algorithm as well as a proof that it produces an optimal solution.