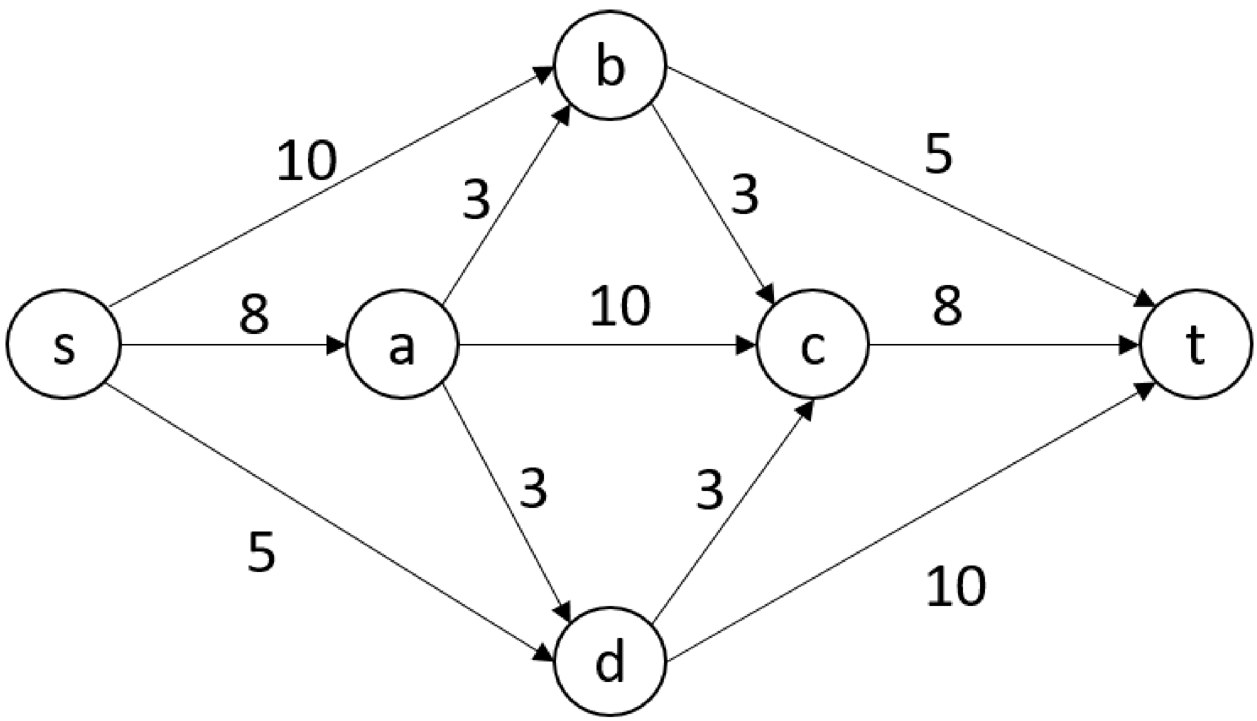
CSCI 3100, Fall 2018

Homework 7 [100 points]

Problem 1

Consider the graph below, where labels on edges represent edge capacity. Write your answers in the space provided.



a. **[25 points]** Manually run Edmonds-Karp algorithm on this graph and determine the maximum flow function. Label the flow on each edge. For example, if max flow algorithm results in 10 units flowing through edge (d, t), label edge (d, t) with 10/10: the first number is the flow you calculated, the second number is capacity provided.

b. **[5 points]** What is the ***value*** of the flow you calculated.

c. **[10 points]** What is the capacity of the minimum s-t cut of this graph?

Problem 2 [30 points]

Consider the following problem. Suppose we have a set of N students $\{s_1, s_2, \ldots, s_N\}$ and a set of M universities $\{u_1, u_2, \ldots u_M\}$. Each university u_i, offers a set of courses C_i and has a limit L_i on the number of students it can accept. Each student s_k wants to take a set of courses T_k. We need to determine whether it is possible to assign students to universities such that:

- Each student is assigned to exactly one university
- Each university u_i is assigned at most L_i students
- Each student is assigned to a university that offers all the courses the student wants to take.

Provide an algorithm for solving this problem (hint: map this problem to a network flow problem). Please type up your solution. You can hand draw any graphs you want to include.

Problem 3 [15 points]

Decide whether you think the following statement is true or false. If it is true, give a short explanation. If it false, give a counterexample.

Let G be an arbitrary flow network, with a source s, a sink t, and a positive integer capacity c_e on every edge e. If f is a maximum s-t flow in G, then f saturates every edge of of s with flow (for all edges e out of s, we have $f(e)=c_e$).

Problem 4 [15 points]

Decide whether you think the following statement is true or false. If it is true, give a short explanation. If it is false, give a counterexample.

Let G be an arbitrary flow network, with a source s, a sink t, and a positive integer capacity c_e on every edge e. Let (A, B) be a minimum s-t cut with respect to these capacities $\{c_e : e \in \mathrm{E}\}$. Suppose we add 1 to every capacity. Then (A, B) is still a minimum s-t cut with respect to these new capacities $\{1 + c_e : e \in \mathrm{E}\}$.