Network Flow

CSCI 3100

What is Network Flow?

Network Flow is a subsection of graph theory related specifically to situations where something moves from one location to another.

An easily visualized example of network flow is piping system through which a quantity of water must pass.

Concepts in Network Flow

"Sources" are nodes which supply a commodity

"Sinks" are nodes which use up a commodity

An edge's capacity is the maximum amount of flow which can pass through it

Graphs are usually directed

Properties/Constraints

A **flow** in a graph G = (V, E) is a function f: V x V -> $\mathbb{R}$ that satisfies the following properties

Capacity constraint: for all (u, v) in V, 0 ≤ f(u, v) ≤ c(u, v), where c(u, v) is a capacity of edge (u, v)

Flow conservation: flow into a vertex equals to flow out of that vertex, with the exception of source and sink vertices.

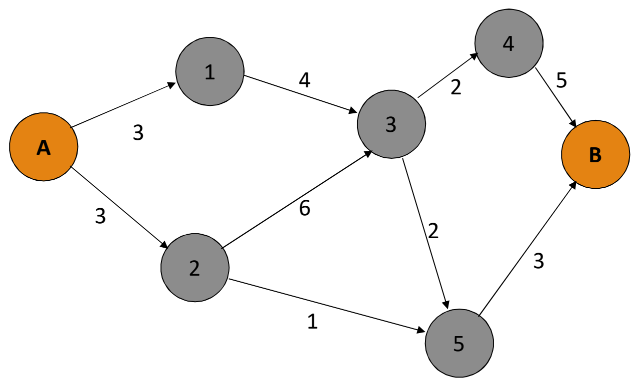
$\sum_{v \in V} \boldsymbol{f(v, u)} = \sum_{v \in V} \boldsymbol{f\,(u, v)}$

G is directed

In-degree of the source vertex is 0

Out-degree of the sink vertex is 0

In the graph below, edge weights are capacities. Let f be defined as:
f(1, 3) = 2
f(2, 3) = 2
f(3, 4) = 3
f(3, 5) = 2
Is this a valid flow function?



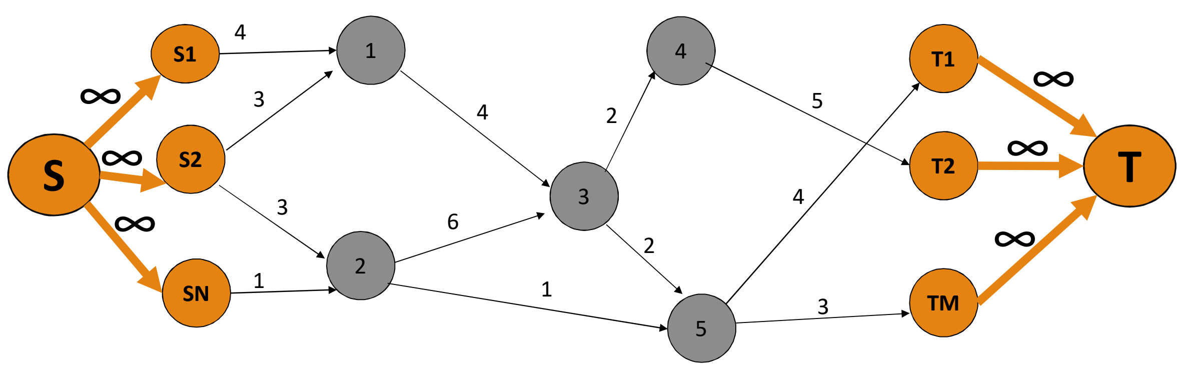
A. Yes

B. No, because it violates capacity constraint

C. No, because it violates flow conservation property

D. B and C

Multiple source and sink vertices



Applications of Maximum-Flow

Homework assignment

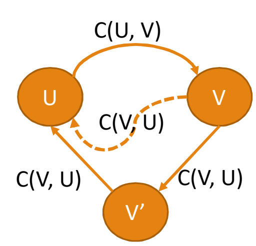
Antiparallel edges

In a flow network, if we have an edge (u, v), we don't want an edge (v, u).

What if we have a network that does not satisfy this constraint?

We can still model this problem as a "flow network":

- If edges (u, v) and (v, u) are present, introduce a new vertex v' and edges:
- (v, v') and (v', u)
- The capacity of (v, v') and (v', u) are the same as capacity of (v, u)
- Remove edge (v, u)

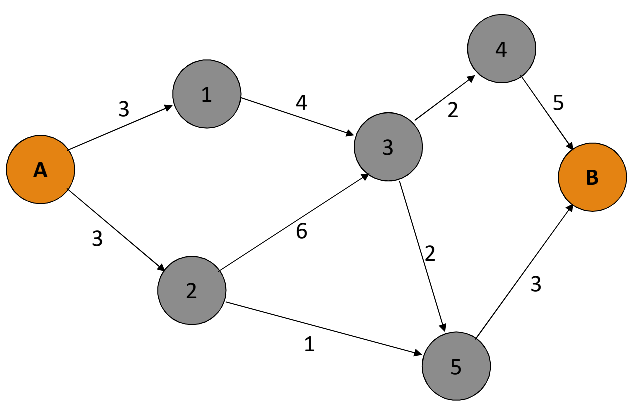


Maximum Flow Problem

Given a graph G = (V, E) where S is the source vertex and T is the sink vertex, find a flow function f, such that $\sum_{v \in V} f(S, v)$ is maximized.

Value of MAX flow

Given the graph below where edge weights are capacities, what is the maximum flow from A to B?



A. 5

B. 6

C. 8

D. 1

E. None of the above

The Ford-Fulkerson method

This is a relatively simple way to find maximum flow through a network:

- Find an unsaturated path from the source to the sink
- Add an amount of flow to each edge in that path equal to the smallest capacity in it
- Repeat this process till no more paths can be found
- The total amount of flow added is then maximal