Maximum Flow Continued

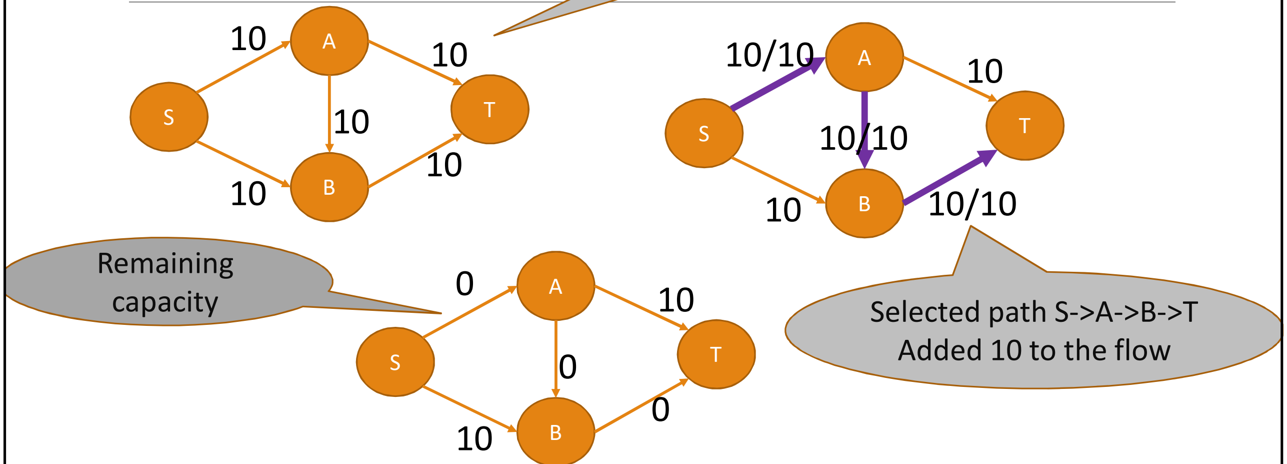
CSCI 3100

Recall the Ford-Fulkerson method

This is a relatively simple way to find maximum flow through a network:

- Find an unsaturated path from the source to the sink
- Add an amount of flow to each edge in that path equal to the smallest capacity in it
- Repeat this process till no more paths can be found
- The total amount of flow added is then maximal

The Unlucky Case



Need the “undo” button

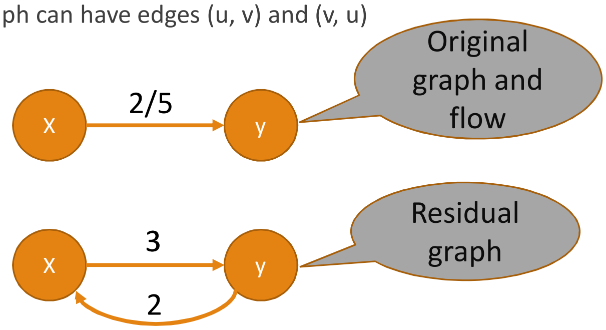
Residual Graph – built from original graph

Edge weights – residual (remaining) capacity

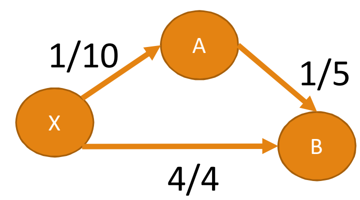
If edge (u, v) is in the original graph, residual graph can have edges (u, v) and (v, u)

Residual capacity is defined as:

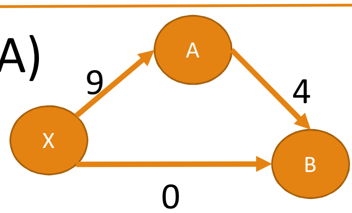
$$c_f(x,y) = \begin{cases} c(x,y) - f(x,y) \ \ if \ (x,y) \in E \\ f(y,x) \ \ if \ (y,x) \in E \\ 0 \ otherwise \end{cases}$$



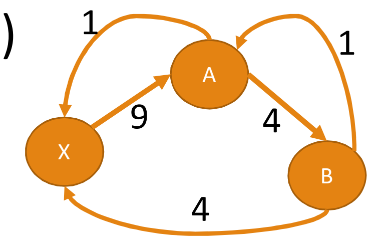
What is the residual graph of this network flow?



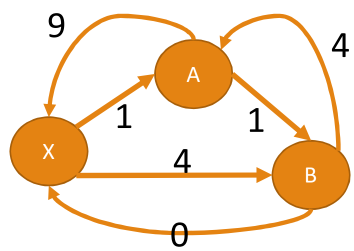
(A)



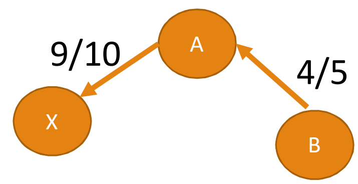
(B)



(C)



(D)



Residual Network

Let G=(V, E) be a directed graph with flow f(u,v) for each edge (u, v) in the graph

Then $G_f = (V, E_f)$ is the residual network of G with flow f, where $E_f=\{(u, v) \in V \times V: c_f(u, v) > 0\}$

Each edge of residual network can admit flow > 0

$|E_f| \leq 2|E|$

We can define flow through residual network, that satisfies

- Capacity constraint with respect to capacities of edges in Gf
- Flow preservation property

Augmentation of flow

Add flow to the original network by defining flow through the residual network

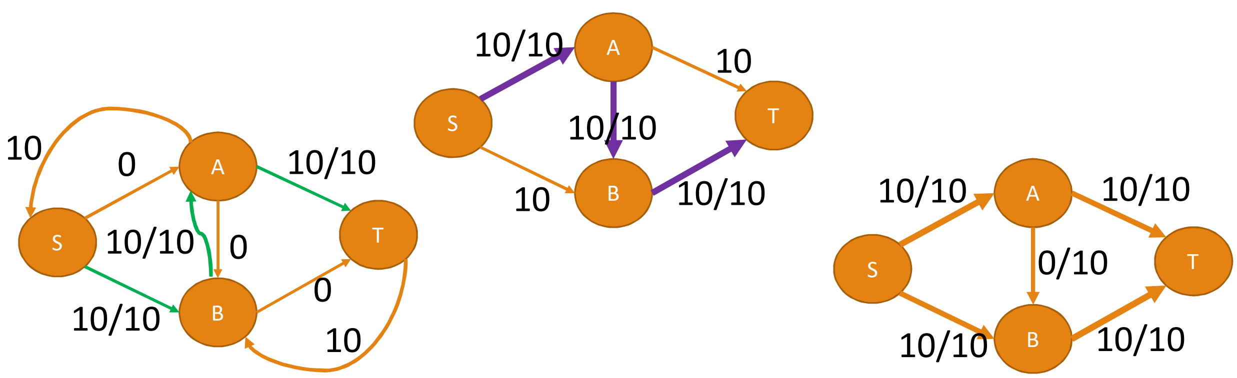
Let f' be a flow through the residual network

If (x, y) ∈E, the augmented flow through edge (x, y) is f(x, y) + f'(x, y) – f'(y, x)

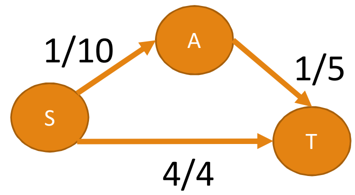
If (x, y) ∉E, the augmented flow through edge (x, y) is 0

Augmentation of f by f

$$f \uparrow f'\ (x, y) = \begin{cases} f(x,y) + f'(x,y) - f'(y,x) \ \ if\ (x,y) \in E \\ 0\ \ otherwise \end{cases}$$



Let f’ be defined as:
f’(S, A) = 2
f’(T, S) = 2
f’(A, T) = 2
What is the augmentation of f by f’?



	(S, A)	(A, T)	(S, T)
A.	3	3	2
B.	3	3	4
C.	1	1	4
D.	2	2	4

Lemma 26.1

Augmented flow satisfies capacity constraint

Augmented flow satisfies flow preservation property

Augmenting path

A simple (no cycles) path from S to T in the residual network G_f

We can increase the flow on edge (x, y) of an augmenting path by up to $c_f(x, y)$ without violating the capacity constraint in network G

Residual capacity of an augmenting path p – maximum amount by which we can increase the flow on each edge of p.

$c_f(p) = \min\{c_f(x, y): (x, y) \text{ is on } p\}$