Maximum flow in a network
- Ford-Fulkerson
- Edmonds-Karp

CSCI 3100

Recall: Max Flow-Min Cut Theorem

If f is a flow in a flow network $G=(V, E)$ with source s and sink t, then the following conditions are equivalent:

1. f is a maximum flow in G
2. The residual network G_f contains no augmenting paths
3. $|f| = c(S, T)$ for some cut (S, T) in G

The Basic Ford-Fulkerson Algorithm

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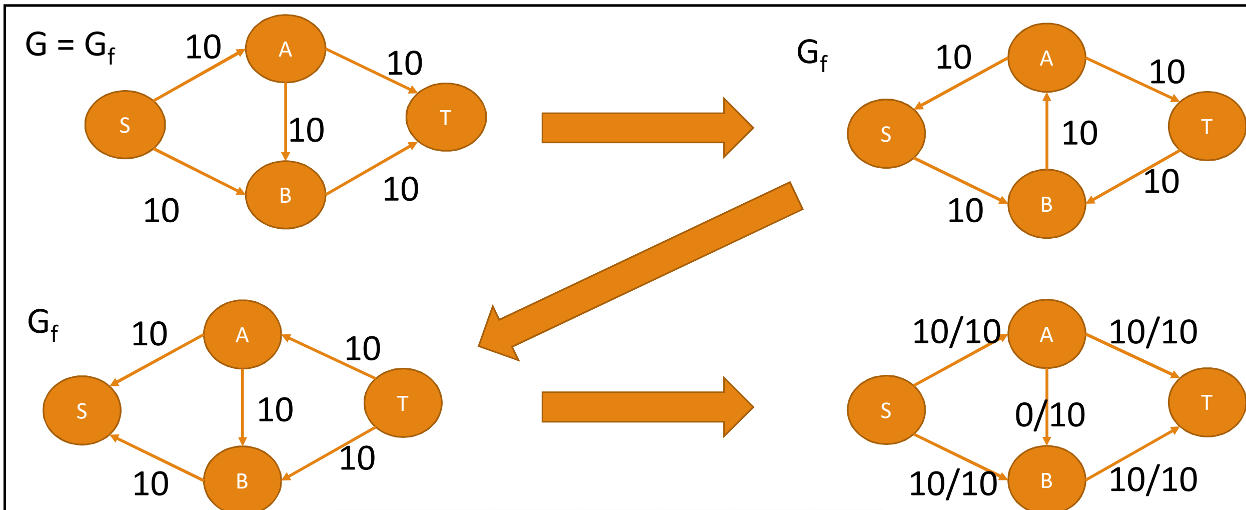
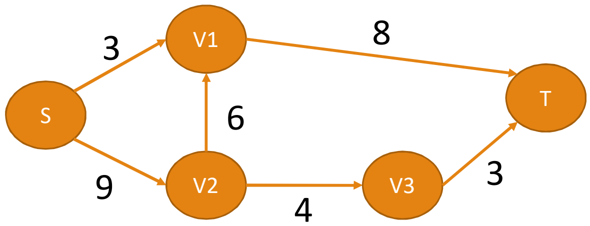
FORD-FULKERSON(G, s, t)
1. for each edge (u, v) in G.E
2.    (u, v).f = 0
3. while there is a path from s to t in G_f
4.    c_f(p) = min{c_f(u,v): (u, v) is in p}
5.    for each edge (u, v) in p
6.       if (u, v) in G.E
7.          (u, v).f = (u, v).f + c_f(p)
8.       else
9.          (v, u).f = (v, u).f – c_f(p)
```



	(S, A)	(S, B)	(A, B)	(A, T)	(B, T)
f	0	0	0	0	0
p=S->A->B->T $c_f(p) = 10$	+10		+10		+10
p=S->B->A->T $c_f(p) = 10$		+10	-10	+10	

Max flow

What is the value of the max flow of the following network (in the picture, edge weights are capacities)?



A. 12

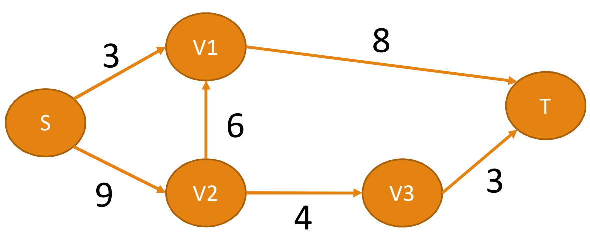
B. 9

C. 11

D. 3

E. None of the above

What is the value of the min cut of the following network (in the picture, edge weights are capacities)?



A. 12

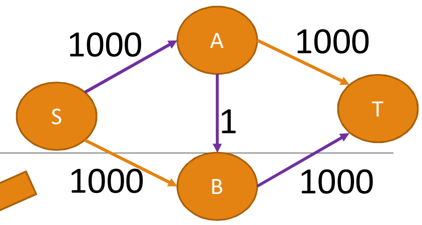
B. 9

C. 11

D. 3

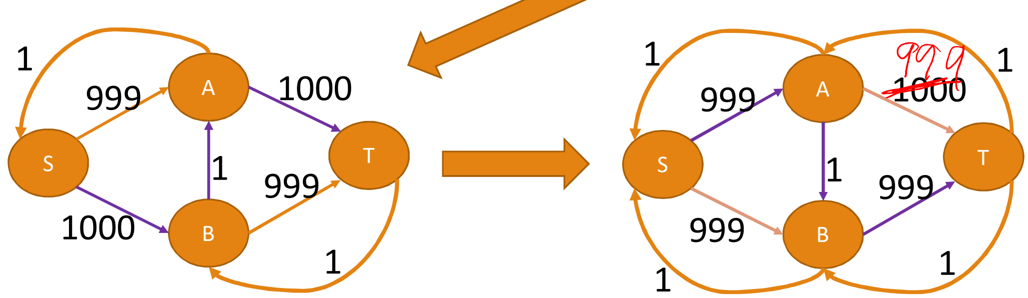
E. None of the above

Run time of the algorithm



Depends on how we select augmenting path

Unlucky case:



Which of the following statements is correct?

FORD-FULKERSON(G, s, t)

1. for each edge (u, v) in G.E
2. (u, v).f = 0
3. while there is a path from s to t in G_f
4. $c_f(p) = \min\{c_f(u,v): (u, v) \text{ is in } p\}$
5. for each edge (u, v) in p
6. if (u, v) in G.E
7. $(u, v).f = (u, v).f + c_f(p)$
8. else
9. $(v, u).f = (v, u).f - c_f(p)$

A. If edge weights are integers, each iteration of the while loop (lines 3 – 9) increases the flow on G by at least 1.

B. We can construct G_f in O(E) time.

C. The search for augmenting path from s to t (on line 3) can be done in O(|V|+|E|) time.

D. If edge weights are integers, and the maximum flow is |f*|, then the algorithm executes in O((|V|+|E|)|f*|).

E. All of the above

Edmonds-Karp Algorithm

Modify Ford-Fulkerson Algorithm:

Use breadth-first search to find augmenting path (line 3 of the Ford-Fulkerson algorithm)

Complexity: $O(|V||E|^2)$

How many times will Edmonds-Karp Algorithm augment the flow f before it finds maximum flow of the network below, where edge weights are capacities (how many iterations of the while loop will be executed) ?

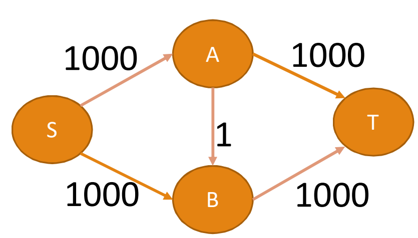
A. 1

B. 2

C. 3

D. 4

E. 20,000



Application of Max-Flow

Determine, whether a given team can have the most wins at the end of a tournament

Input:

$\{t_1, t_2, \dots t_N\}$ – teams

$\{W_1, W_2, \dots, W_N\}$ – number of wins for each team

$R = \{r_{ij}$: # of times teams i and j will play each other in the future$\}$

Calculate $R_1, R_2, \dots, R_N$ – ~~remailing~~ remaining games for each team

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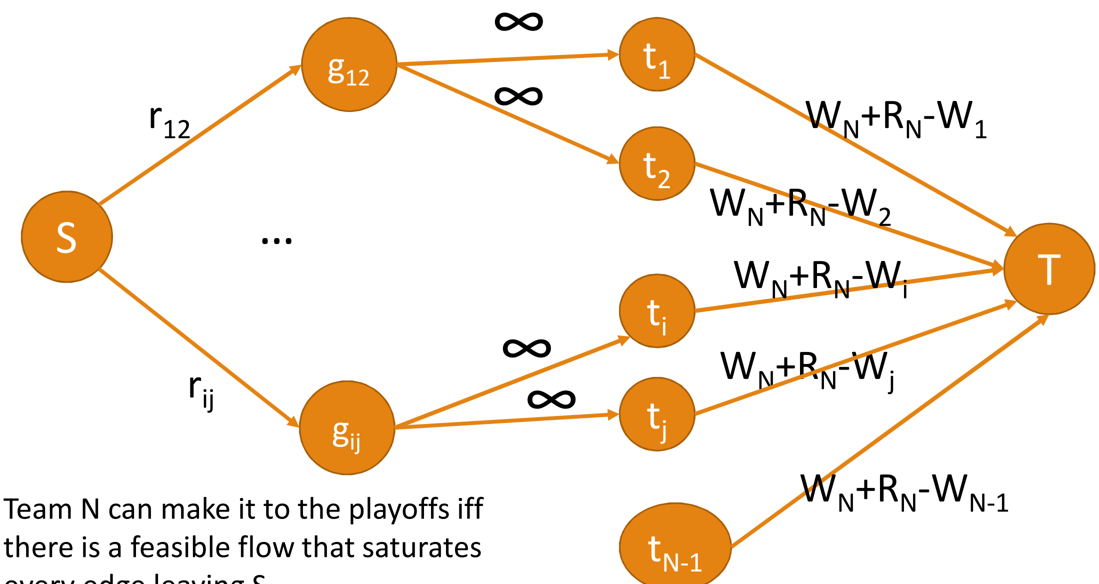
Can team N make it to the playoffs (have the most wins)?



Team N can make it to the playoffs iff there is a feasible flow that saturates every edge leaving S